

MODELLING SURFACE HEAT EXCHANGES FROM A CONCRETE BLOCK INTO THE ENVIRONMENT

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Abstract

The presented problem was to determine an appropriate heat transfer boundary condition at the surface of a concrete slab exposed to the environment. The condition obtained involves solar radiation and convective heat transfer, other terms were shown to be small compared to these. It is shown that this boundary condition leads to a temperature variation that has qualitative agreement with experiments carried out by the Cement and Concrete Institute.

1 Introduction

The long term performance of concrete structures, *e.g.* durability and strength, can be greatly influenced by the initial thermal field, see for example Schindler *et al* (2004), Wojcik *et al* (2003), Yu *et al* (1998). Hence, the removal of heat from concrete during the initial drying stage is essential to minimize undesirable effects. Temperature fluctuations can also be a problem, leading to shrinkage and expansion which in turn may lead to large stresses and cracking, see Liu *et al* (2002).

When concrete is prepared and placed under conditions of high ambient temperature, low humidity, solar radiation or wind, an understanding of the

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effects these environmental factors have on concrete properties and construction operations is required. Once these factors are understood, measures can be taken to eliminate or minimize undesirable effects (Alabidien 1991). To describe the heat loss from a concrete slab into the environment the Cement and Concrete Institute (CCI) fixes the concrete temperature at the surface to the ambient temperature. Experiments have shown that in reality the surface temperature differs significantly from ambient, see Schindler *et al* (2004), Bentz (2000), Gilliland & Dilger (1997). Laboratory experiments reported by the CCI also show that the surface temperature variation is out of phase with the ambient variation. This is confirmed by the results of Gilliland & Dilger (1997) obtained during the construction of a bridge in Canada.

In this paper we attempt to improve on the current boundary condition and explain both the temperature difference and phase lag.

2 Modified boundary condition

We will consider a one-dimensional model to simplify the analysis and allow us to focus on the boundary condition. Physically this corresponds to a block with a large top surface area (this will be discussed later) and we only take temperature readings away from the edges of the block. However, this one-dimensional analysis is illustrative. In reality a block losing heat from all surfaces would require the same boundary condition applying to each surface.

In one dimension the heat flow is modelled by

$$\rho_c c_c \frac{\partial T}{\partial t} = \kappa_c \frac{\partial^2 T}{\partial z^2}, \quad (2.1)$$

where ρ_c, c_c and κ_c are the density, specific heat and conductivity (respectively) of concrete. This may be solved subject to a fixed temperature at the base $T = T_g$, where T_g is the ground temperature. This could be taken equal to the ambient temperature. Initially, when the concrete has been poured, we can assume the temperature is constant throughout the block $T(x, 0) = T_0$. At the top surface, which is in contact with the air, the current boundary condition used by the CCI is

$$T = T_a(t)|_{z=L}, \quad (2.2)$$

where T_a is the ambient temperature. The ambient temperature may be

approximated by

$$T_a = \left(\frac{T_{max} + T_{min}}{2} \right) - \left(\frac{T_{max} - T_{min}}{2} \right) \sin \left(\frac{2\pi(t + t_w)}{24 \times 3600} \right), \quad (2.3)$$

where t_w is the time at which the temperature is a minimum. This occurs usually about an hour before sunrise. In the following we assume that the temperature is a minimum at 5.30am which implies that $t_w \approx 5.5 \times 3600$ sec. Obviously a more accurate expression could be obtained through local meteorological data.

Equation (2.2) states that there is continuity of temperature across the air-concrete interface. Anybody who has witnessed hot air rising from a road or other such surface will know that this is not the case in reality. A more accurate condition requires continuity of energy flux. To use such a condition we must first determine the appropriate energy source and sink terms. The standard terms are

$$Q_{sun} = \gamma_{abs} Q_{inc}, \quad Q_r = \epsilon \sigma (T^4 - T_a^4), \quad Q_h = H(T - T_a),$$

where Q_{sun} represents the contribution from the sun shining on a surface, Q_r represents radiation from the slab to the sky and Q_h is convective heat transfer. The constant $\gamma_{abs} = 0.65$ is the solar absorptivity of concrete, $Q_{inc} \sim 700 \text{W/m}^2$ is the solar radiation normal to the surface and Q_{sun} is zero at night time. In the radiation expression, $\epsilon = 0.9$, $\sigma = 5.7 \times 10^{-8} \text{W/m}^2 \text{ } ^\circ\text{C}^4$. If $T_a > T$ radiation contributes to the heating, if $T_a < T$ then heat is being lost by radiation to the sky. The convective heat transfer coefficient H needs to be determined experimentally, however simple formulae are reported in Bentz 2000.

With these energy terms the appropriate boundary condition at the exposed surface $z = 0$ is

$$-\kappa_c \frac{\partial T}{\partial z} = Q_{sun} f(t) + Q_r + Q_h = Q_s + \epsilon \sigma (T^4 - T_a^4) + H(T - T_a), \quad (2.4)$$

where $f(t)$ is simply a switch function to turn off the solar radiation term at night. For example $f(t) = 1$, $t \in [6, 18]$ hrs, $f(t) = 0$, $t \in [18, 6]$ hrs.

If the ambient temperature is $T_a = 20^\circ\text{C}$ then we require $T \sim 100^\circ\text{C}$ for Q_r to be of the same magnitude as the other terms. When T takes a more typical value, $T \sim 60^\circ\text{C}$ then $Q_r/Q_h \sim 10^{-2}$ and so Q_r may therefore be neglected in all realistic calculations. A wet concrete slab would also lose energy due to evaporation. This was estimated during the study week to be negligible compared to other terms. However, it is retained in certain published models, see Schindler (2004) for example.

A realistic boundary condition to replace $T = T_a$ at the surface is therefore

$$-\kappa_c \frac{\partial T}{\partial z} = Q_s + H(T - T_a) . \quad (2.5)$$

This is a standard cooling condition. When the heat transfer between the air and the concrete is large, $H \rightarrow \infty$, it reduces to the condition currently used by the CCI. However, in general this condition can lead to significantly different results to those obtained by setting $T = T_a$.

3 Verification of the new boundary condition

Now consider the laboratory experiment reported by the CCI. The results showed a temperature variation on the concrete surface less than that of the ambient temperature and with a time lag.

To describe the experiment we first scale the governing equation, equation (2.1). With length-scale L and a time scale of one day, $\tau = 24 \times 3600\text{s}$, we obtain

$$\frac{\rho_c c_c L^2}{\kappa_c \tau} \frac{\partial T}{\partial t'} = \frac{\partial^2 T}{\partial z'^2} , \quad (3.1)$$

where $z' = z/L$, $t' = t/\tau$. We note that the diffusion length-scale

$$L = \sqrt{\kappa_c \tau / \rho_c c_c} \approx 0.24\text{m} ,$$

that is, over the period of one day we expect the energy loss from the surface to have an effect over distances of the order 20cm. Gilliland & Dilger (1997) give data for temperature profiles at depths 7.5, 27.5 and 157.5cm inside a concrete block which is exposed to the atmosphere. The ambient variation is closely mimicked by the temperature profile at 7.5cm (with a time lag). At 27.5cm the variation is barely noticeable and at 157.5cm the ambient variation has no effect. Hence the length-scale of 20cm appears entirely reasonable. Effectively this means that the bottom of a standard block is sufficiently far from the surface for it to notice the daily variation. Further, to carry out experiments using a one-dimensional model we would require a block with a top surface significantly larger than $20 \times 20\text{cm}^2$. We therefore solve the system

$$\frac{\partial T}{\partial t'} = \frac{\partial^2 T}{\partial z'^2} , \quad (3.2)$$

$$-\frac{\partial T}{\partial z'} = \Lambda(T - T_a) \Big|_{z'=0}, \quad \frac{\partial T}{\partial z'} = 0 \Big|_{z' \rightarrow -\infty}, \quad T(z', 0) = 20, \quad (3.3)$$

where

$$\Lambda = LH/\kappa_c = H\sqrt{\tau/\rho_c\kappa_c}. \quad (3.4)$$

Note solar radiation is neglected since the experiment is indoors. We also replace $T = T_g$ with a zero flux condition, since, as mentioned, the base in this scaling is a long way from the area of interest.

Numerical solutions for this system are shown in Figures 1–3 for a 10m deep block initially at 20°C up to time $t = 72$ hours. With a length-scale $L = 0.24\text{m}$ this means the domain extends for $z \in [0, 41.7]$. The ambient temperature varies according to equation (2.3), with $T_{max} = 30$, $T_{min} = 10^\circ\text{C}$.

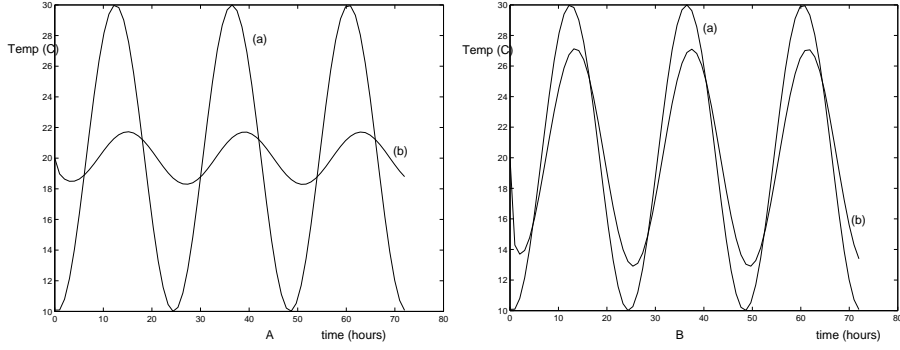


Figure 1: Temperature variation for 3 days, at the top surface of the concrete with A) $\Lambda = 0.1$, B) $\Lambda = 1$ and boundary conditions a) ambient temperature, b) cooling condition.

It is clear from Figures 1 A) and B) that the new boundary condition exhibits the required behaviour. On figure 1 A) the numerically calculated temperature depicted by curve (b) varies between 16.5 and 23.5°C and lags behind the ambient temperature. The form of the numerical curve depends on the single parameter Λ . Increasing Λ increases the correspondence between the curves, both the amplitude increases and the time lag decreases. This may be observed on Figure 1 B) which has $\Lambda = 1$. The condition $T = T_a$ is therefore reasonable (in the absence of sunlight) for high Λ . The amplitude difference and time lag therefore decrease with an increase in H or τ and a decrease in ρ_c, κ_c . Note, there is a square root dependence on

all parameters except H . The variation with time-scale τ indicates that a long period variation, such as seasonal changes, do not have a great effect on results obtained via the two conditions. However, short period variations, of the order of days, lead to a significant difference in the results.

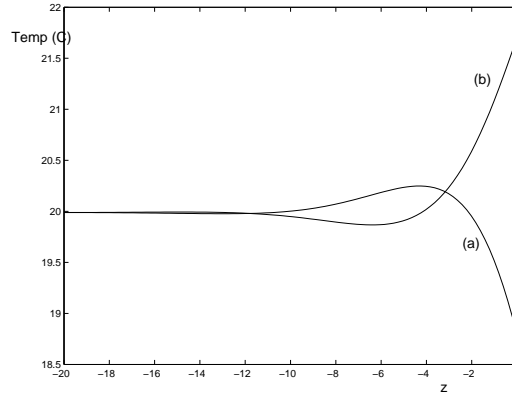


Figure 2: Temperature throughout the concrete at a) $t = 72$ hours, b) $t = 63$ hours.

Figure 2 shows the temperature from the surface to a distance $20L$ inside the block at times $t = 63, 72$ hours. When $t = 72$ hours the block is coolest at the surface and the temperature increases as z decreases up to a maximum, it then starts to decrease again. This is because the cooling from the outside has not reached all the way in and parts of the block are still feeling the effect of an earlier heating period. For $t = 63$ the opposite occurs, *i.e.* the block is hottest at the outside, cools inwards to a minimum and then increases again.

Figure 2 also gives us a further indication about the length-scale over which temperature variations occur. The calculated length-scale L indicates the order of magnitude over which significant temperature changes are expected in the stated time-scale of 1 day. This does not mean that at a distance L from the surface a numerical calculation should stop. In fact, from Figure 2 it is clear that as far back as $10L \sim 2.4\text{m}$ (corresponding to $z = -10$ on the figure) differences can be observed in the two solutions. However, these differences are a result of heating and cooling over 3 days and so the temperature variation has penetrated much further than the length-scale implies.

Finally in Figure 3 we show the full temperature profile variation through-

out the block for 3 days.

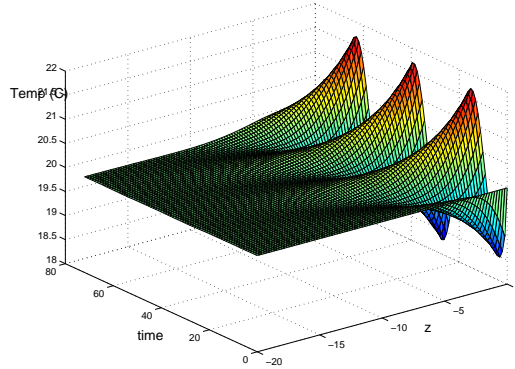


Figure 3: Temperature throughout the concrete for $t \in [0, 72]$ hours.

4 Conclusions

The cooling condition (2.5) appears to be a sensible replacement for the current condition. Results show that the smaller than ambient temperature variation and time lag observed in experiments are reproduced.

Of course there remains a great deal to be done with this work. In particular, the heat transfer coefficient H is always problematic to pin down. It's value depends on a number of parameters, including wind speed, surface roughness and orientation of the slab (dam walls are usually almost vertical and have a lower heat loss rate). Tables and formulae for H do exist for various surfaces and wind speeds, however, in practice, it may be sensible to carry out an experimental study to determine H for specific conditions.

The example shown was aimed at modelling a one-dimensional laboratory experiment. In this case the solar radiation was neglected and so the model depended on a single parameter Λ , see (3.4). The numerical results showed that increasing Λ produced results closer to those obtained by setting $T = T_a$. This is to be expected since in the limit $H \rightarrow \infty$ the new boundary condition reduces to $T = T_a$. The addition of solar radiation to the model will significantly change results since the solar radiation term is of the same order of magnitude as the heat transfer term.

Finally, whilst the model has been shown to exhibit the correct qualitative behaviour, it must be tested much more thoroughly. To properly verify

this model it should be:

- a) compared against data from the laboratory experiments
- b) compared against one-dimensional data made outside the laboratory (so including the solar radiation term).

A study of the heat transfer coefficient must also be carried out for various realistic situations.

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