Pilkington Contact Lenses

The Study Group was asked to investigate

(i) the mechanism for centring the lens on the cornea,

(ii) the movement of the lens during blinking—observed to be a nett upward movement of 2 mm,

(iii) the thickness of the thin film of tear fluid which covers the eye – observed to be 10 μm,

(iv) the transport of oxygen, nutrients and debris under the lens.

The tear fluid was supposed to have a viscosity about five times that of water, i.e., 5×10^{-3} Pa s, and a surface tension of about half that of water, i.e. 4×10^{-2} N m^{-1}. The Study Group concentrated attention on the small hard (gas permeable) lenses which are about 0.1mm thick and 8 mm in diameter (so fitting on the cornea).

Lubrication results

(The Study Group enjoys lubrication problems, and will turn a problem into a lubrication problem if it possibly can. So, here are some lubrication results—they might just be relevant.)

Draining of a thin film on a flat vertical plate. For a film of thickness h moving at speed U, a balance of the viscous stress $\mu U/h$ with the hydrostatic pressure $\rho g h$ yields a film velocity $U = \rho g h^2/\mu$ which is 0.2 mm s^{-1} for a 10 μm film. Thus the tear film will drain the height of the eye, 1 cm, in 50 s. Now one blinks about four times a minute in order to maintain the film. Clearly a thicker film would require more rapid blinking, with frequency proportional to h^2.

A flat lens sliding down a flat vertical plate. The 100 μm thick lens increases the weight sliding down by a factor of 10. Thus the lens will slide down at 2mm s^{-1}, and so should fall off a 1 cm eye in 5 s. This simple calculation would seem to suggest some active support of the lens is necessary. A complication, however, is any rotation of the lens. The tangential frictional force on the lens will make it rotate with the top coming away from the plate. The lens will then be sliding down in a configuration opposite to a classical thrust bearing, and so the bottom of the lens will be sucked even closer to the plate. This orientational instability might make the sliding cease quickly on rough plates. (How far would the lens slide down before making contact with the plate?) This may be the explanation of why a lens did not slide off a glass marble in some experiments in Oxford. The eye is likely, however, to be much smoother than the marble!

The squeeze film for a lens settling onto a horizontal plate. The standard analysis for a disk of radius a and thickness d squeezing out a film of thickness $h(t)$ has a force balance $\rho \pi a^2 d g = 3\pi\mu \dot{h} a^4/h^3$. Thus the time to settle is $3\mu a^2/h^2 \rho d g$, which for the lens would be 2 hours.

The capillary forces keeping the lens on. In order to pull the lens off, the interface between the air and tear fluid must come under the lens. When the interface first comes under the lens it will have a radius of curvature equal to the thickness of the film, i.e. $10\,\mu$m. Now the ratio of the resulting surface tension pressure γ/h to the hydrostatic pressure under the lens $\rho g d$ is about 4×10^3. Thus to pull the lens off one must exert a force equal to the weight of a lens 4×10^3 thicker, i.e. 40 cm thick. Alternatively, the capillary forces will pull the lens down so that the film wets the entire underside of the lens in 4×10^3 faster than the gravity settling time, i.e. 1 second.

Centring

Capillary forces. A literature search before the Study Group turned up the following idea. Fluid from the meniscus at top of the lens drains to the meniscus at the bottom. The curvature of the top meniscus is thus increased and that of the bottom decreased. Hence there is a lower pressure in the tear film at the top of the lens compared with the bottom. This pressure gradient drives a flow upwards in the lubrication film behind the lens. The associated upward tangential viscous force supports the weight of the lens. This argument has two problems. First, it does not address the centring issue—it would apply equally to a flat plate. Second, the flow will try to eliminate the difference in the curvature of the menisci—there is a risk of this argument producing a perpetual motion machine! The capillary forces are, however, capable of easily supporting the weight of the lens ($2\gamma a$ exceeds $\pi a^2 d \rho g$ by a factor of 10, n.b., less than the earlier 4×10^3), although it might be necessary for the lens to be partly dry in order to obtain the maximum upward capillary force $2\gamma a$.

Geometry. Some experiments (at lunchtime) sliding an upside down cup (the lens) over an upside down saucer (the cornea) on an upside down plate (the eyeball) soon convinced everyone that the lens cannot move off the central position of maximum curvature without one edge separating from the eye. The lens has a diameter of 8 mm and fits the cornea, which has a radius of curvature of 8 mm. The white of the eye, however, has a radius of curvature of 16 mm. Thus when the lens comes off the cornea there should be a 0.5 mm gap between the white of the eye and the lens. This is not observed. Moreover, there is insufficient tear fluid to fill such a wide gap.

What in fact happens is that the *eyeball deforms* to fill the gap. The eye can be considered as a fairly floppy bag inflated to a pressure of about $\frac{1}{100}$th of an atmosphere, i.e. 10^3 Pa. The force from this pressure over the area of the lens when it is off the cornea would then be 5×10^{-2} N, i.e. about a quarter of the capillary forces which hold the lens on (but a thousand times the weight). When the lens does come off the cornea and is completely on the white of the eye, then it observed to slide downwards fairly quickly. Thus an active centring is required to keep the lens on the cornea.

The centring mechanism seems to be the deformation of the eyeball. The lens can slide round the cornea because they have the same curvature. When the lens comes to the edge of the cornea, the eyeball (probably muscle there) must deform there. This deformation will be resisted by a force which will flip the lens back onto the cornea.

The above centring mechanism suggests a way of maintaining the orientation of the lens for treatment of astigmatism. If the lens is made a little smaller than the cornea and is not circular but has a flatter bottom edge, then it might 'rest' on the eyeball with a preferred orientation.

Blinking

The study group did not have much time to consider the movement of the eyelid and was unsure of the geometry of the lid. If the lid is considered as a flat plate moving at U across a flat eye with a thin lubricating layer h of tear fluid between, then a flux of $\frac{1}{2}Uh$ means that half the film is not transported with the lid. Thus if the lid is to leave a $10\,\mu$m layer when it retreats it must itself be $20\,\mu$m above the eye. It is not clear how the lid can advance over the eye without running dry of fluid.

While not answering the above question, it is almost certain that the lid is closer to the lens when it retreats compared with its advance. This would mean on a flat plate that the lens is dragged up more than it is dragged down. It is more likely, however, that the excursion of the lens is governed by the centring mechanism. Note that the lid moves about five times faster than the speed calculated earlier the fall of a lens under its own weight. Thus the lid is likely to exert a tangential force five times greater than the weight of the lens.

The Study Group had no time to consider the transport in the film under the lens.

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