Evaluation of Customer Lifetime Value

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Suggested Model (Berger & Naar)

\[ CLV = C \sum_{i=0}^{n} \frac{r^i}{(1 + d)^i} - M \sum_{i=1}^{n} \frac{r^{i-1}}{(1 + d)^{i-0.5}} \]

* Applicability?
  - Not in current form!
* Modified Model?
  * Yes! For 1 customer in 1 service:

\[ CLV = \sum_{i=1}^{n} \frac{C_i \prod_{j=1}^{i} r_j}{(1 + d)^i} - \sum_{i=1}^{n} \frac{M_i \prod_{j=1}^{i} r_j}{(1 + d)^{i-0.5}} \]

\[ \text{set } \prod_{j=1}^{m} r_j = 1 \text{ if } m < 1 \]

\( C_i = i^{th} \) month's gross contribution
\( M_i = i^{th} \) month's marketing spend
\( r_i = i^{th} \) month's retention rate
\( n = \) number of months, \( d = \) discount rate.

Determining the retention rate, \( r_i \).
Proposed method:
use the recursion formula

\[ r_{i+1} = 1 - (1 - r_i)e^{-kM_i} \]

Assumption: Marketing in the current month effects retention this month!

Now we need an initial value, \( r_1 \).
We have 2 suggestions.
Both base the value for any given customer on a group of "similar" customers.
The value is calculated for the group and is applied to each individual with those characteristics.

One is simple but under-estimates retention.
Suppose we have historical data of account closures for up to \( N \) months. Let \( n_i \) be the number of accounts that closed after \( i \) months.

\[
\begin{array}{ll}
\text{months} & \text{customers} \\
1 & n_1 \\
2 & n_2 \\
3 & n_3 \\
\vdots & \vdots \\
N & n_N \\
\end{array}
\]

\[P(j) = \frac{n_j}{(\sum_{i=1}^{N} n_i) - (\sum_{i=1}^{j-1} n_i)} = \text{prob (leave in month } k \text{ | survived } k-1 \text{ months)}\]

\[r_i = 1 - \frac{\sum_{j=1}^{N} P(j)n_j}{\sum_{j=1}^{N} n_j}\]

weighted average on number of customers.

This should take account of any lock-in period.

The other takes currently active customers into account.

We wish to establish the relationship between the data & the model.

Model:
\[
\begin{align*}
\text{Prob( Account lasts through the } 1^\text{st} \text{ month)} &= r \\
\text{Prob( Account lasts through 2 months)} &= r^2 \\
\vdots \\
\text{Prob( Account lasts exactly 1 month)} &= 1 - r \\
\text{Prob( Account lasts exactly 2 months)} &= r(1 - r) = r - r^2 \\
\text{Prob( Account lasts exactly 3 months)} &= r^2(1 - r) = r^2 - r^3 \\
\vdots
\end{align*}
\]

This relates to closed accounts.

Whence: \( \text{Prob ( account closes BEFORE reaching } p \text{ months)} \)

\[
= \sum_{i=1}^{p-1} r^{i-1}(1 - r) \\
= (1 - r) + (r - r^2) + (r^2 - r^3) + \ldots \\
= 1 - r^{p-1}
\]

\[\Rightarrow \text{Prob(Account is still active in } p^{\text{th}} \text{ month.)} = r^{p-1}\]

This relates to active accounts.
What is the overall probability that we see the data we have if it obeys the model we impose?

\[ p = (1 - r)^{n_1} [r(1 - r)]^{n_2} \cdots (r^2)^{n_3} \cdots \]

where

\[ n_1 = \text{no of accounts still active & only established for 1 month} \]

\[ n_2 = \text{no of accounts still active & only established for 2 months} \]

We now choose \( r \) to maximize \( P \).

This is where the relationship between the data & the model is strongest.

PCCW’s next question was about combining CLV’s to get the overall CLV.

Assume there are \( K \) services:

\[ CLV_{TOTAL} = \sum_{k=1}^{K} CLV_k, \quad \text{where...} \]

\[ CLV_k = \sum_{i=0}^{n_k} C_{ik} \prod_{j=1}^{i} r_{jk} \left(\frac{1}{1+d}\right)^i - \sum_{i=1}^{n_k} M_{ik} \prod_{j=1}^{i} r_{jk} \left(\frac{1}{1+d}\right)^{i-0.5} \]

Where \( M_{ik} \) is a function of \( r_{i-1k} \) but this is company Marketing policy.

and \( C_{ik} = \text{Revenue}_{ik} - \text{Cost of Sales}_{ik} \)

Now \( \text{Revenue}_{ik} = \text{Usage}_{ik} \times \text{unit price}_{ik} \)

Let \( \text{UP} = \text{unit price} \) & \( \text{US} = \text{usage} \).

Unit price is determined by company strategy and should be kept separate from other Marketing costs, \( M_{ik} \).

We suggest that usage\(_{ik} \) is a function of unit price\(_{ik} \)

\( M_{ik} \) and cross-effects of advertising \( M_{ij} \times M_{ik}, j \neq k, j = 1, \ldots , k \).

\[ US_{ik} = f_1(M_{ik}) + f_2(M_{ij} \times M_{ik}) + f_3(\text{UP}_{ik}) \]

again \( j \neq k, j = 1, \ldots , k \). Now

\[ f_1 = \frac{L}{1 - c_1 e^{-rM_{ik}}} \]

\[ L = m_k US_{max} \]

where \( m_k \) needs to be estimated by PCCW & depends on expandability of service

\[ US_{i-1k} = \frac{L}{1 - c_1} \]

\( r \) is estimated from historical data. \( f_3 \) is the price sensitivity.

PCCW may already know this!

If not, we have a couple of suggestions ... .

eg \( f_3 = a \text{UP}_{ik} + b \)

so \( b = \text{maximum usage}_k - US_{i-1k} \)

\[ 0 = a(\text{UP}_{i-1k}) + b \]

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Alternatively ... price sensitivity is proportional to the slope. Finally $f_{2j}$ could be given the form: $f_{2j} = c_2(M_{ij} \times M_{ik})^2$ & $c_2$ is determined from historical data.

The fourth PCCW question was about optimizing the Marketing strategy to improve profits. Now $CLV_{TOTAL}$ can be optimized with respect to $M_{ik}$ subject to $\sum_{i=1}^{n} \sum_{k=1}^{K} M_{ik} = \text{Budget}$.

**SUMMARY.**

- Total CLV can be optimized against the promotional costs associated with all PCCW services.
- Price sensitivity is built into the model.
- Retention rates are now a function of the promotional costs.
- Cross-effects of advertising are taken into account through the usage function.
- Unit price of each PCCW service is now in the model and is separate from the other promotional costs.
- The impact of unit price on usage is also built into the model.
- The new model can revert to the original model if $M_{ik}, C_{ik}$ and $\tau_{ik}$ are all set to be constants $M, C$ & $\tau$. 