OPTIMISED DRAGLINE PLANNING MODEL

Pacific Coal provided data for typical operating parameters used in dragline operation. The problem for the Study Group was to investigate whether an optimal model of dragline operation could be developed.

The Study Group modelled the sequence of operations for a typical surface mining strip. Overall, a simulation approach seems necessary to fully represent the dragline operation. Some aspects of the operations that are amenable to optimisation are described in this report.

1. Introduction

In Australia, approximately 12% of total foreign earnings come from exporting coal. The walking dragline is used extensively to remove overburden in the surface mining of coal in Australia. A total of around 60 dragline machines are used to strip overburden from above about 35% of the coal mined in Australia.

It is estimated that an improvement of 1% in the efficiency of dragline operation would contribute an extra $35 million to Australia's export earnings.

There are a number of ways that draglines can be used to remove the overburden from above a coal seam. Figure 1 shows the method considered here.

Figure 1: Overview of Dragline Operation
The dragline sits on an operating bench which is cut below the surface level.

A given dig cycle in the process of mining the coal involves the excavation of three blocks. The overburden and bridge blocks are removed to expose the next section of coal. Since the bridge is built using material from the previous cycle, its removal involves rehandling the overburden. The highwall bench block is removed to prepare the operating bench level for the dragline to use during the next cycle.

Some of the material removed during the excavation is used to build a new bridge and the rest is dumped onto the spoil pile.

At the end of the cycle the situation will be as in figure 1 but shifted back by the excavation length along the strip.

The general sequence of operations during the cycle for the excavation of the highwall, overburden and bridge blocks is as follows:

1. Starting at position 1, the dragline removes material from the highwall bench.

   The material from the highwall bench block is used to start building the new bridge.

   Once the new bench region is excavated the dragline begins to remove the overburden adjacent to the highwall. As this cut deepens the dragline steps forward to reach the lower material, ending at position 2.

   During the initial cut into the overburden the dragline moves in line with the highwall and digs a narrow trench down to the coal. This initial excavation gives a clean highwall face and is called the keycut.

   At some stage the new bridge will be finished. The material is then dumped to extend the spoil pile.

2. Once the keycut is complete the dragline moves to position 3 and begins to remove the next section of overburden, stepping forward towards position 4 as the depth of the dig increases. When the coal seam is reached and exposed the dragline moves to position 5 and repeats this process for the next section of overburden.

3. This process is repeated until all the material in the overburden and bridge blocks has been excavated and dumped onto the spoil pile.

   The dragline will move out onto the new bridge during the final excavation of the old bridge.

4. Once all the material has been excavated the dragline moves off the new bridge and takes up position ready for the next cycle. This final position is at a distance equal to the length of the excavation along the strip behind the starting position.
Statement of the problem

Given the depth of the coal below the surface and the thickness of the coal seam, and the overburden geology, examine what methods are available to optimise overall performance or aspects of the performance.

2. Factors affecting the excavation time

In order to formulate the problem satisfactorily, we need to look in more detail at the particular operations involved in excavating the overburden.

As a first step we need to split the blocks that are to be excavated into sub-blocks of suitable size. The material excavated from each of these sub-blocks will be dumped onto a corresponding sub-block on either the new bridge or the spoil pile.

We will use the notation $O-D$ to denote the Origin-Destination pairs of sub-blocks that will be excavated and filled during the cycle.

The basic operations in the excavation of a given sub-block are to fill the bucket, to hoist it vertically upwards and to swing over to a position above the associated destination sub-block. Once the contents of the bucket are dumped, the return operations are to swing back and lower the bucket onto the origin sub-block and repeat the process.

For the present study we assume that the bucket filling time and the quantity of material picked up by the bucket are independent of the sub-block location except for the fact that the dragline operates at 70% efficiency when digging material above the operating level.

For a given $O-D$ pair, the time required to hoist, swing over, swing back and lower the bucket depends on:

a) The vertical distance through which the bucket must be moved to allow a clear swing from above the $O$ sub-block to above the $D$ sub-block. This distance will depend on the relative heights of the $O$ and $D$ sub-blocks and on the operating level if it is necessary to lift the bucket clear of unexcavated sections of the overburden and bridge blocks.

b) The angle through which the bucket is moved. This will depend on the relative horizontal co-ordinates of the $O-D$ pair and the dragline position.

The final factor affecting excavation time is the time needed to move the dragline between the dig positions associated with successive $O-D$ pairs.
3. General mathematical formulation of the problem

Parameters

We use the following parameter values.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{down}}$</td>
<td>45 m</td>
<td>Maximum dragline digging depth</td>
</tr>
<tr>
<td>$D_{\text{up}}$</td>
<td>45.7 m</td>
<td>Maximum dragline dump height</td>
</tr>
<tr>
<td>$D_{\text{reach}}$</td>
<td>82 m</td>
<td>Effective dragline reach</td>
</tr>
<tr>
<td>$Y_{\text{ob}}$</td>
<td>40 m</td>
<td>Overburden depth</td>
</tr>
<tr>
<td>$Y_{\text{coal}}$</td>
<td>10 m</td>
<td>Depth of coal seam</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>76°</td>
<td>Highwall bench angle</td>
</tr>
<tr>
<td>$\beta$</td>
<td>38°</td>
<td>Spoil repose angle</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>60°</td>
<td>Keycut and working face angles</td>
</tr>
<tr>
<td>$V_b$</td>
<td>46 m$^3$</td>
<td>Volume of material in the bucket</td>
</tr>
<tr>
<td>$v$</td>
<td>200 m h$^{-1}$</td>
<td>Walking speed of the dragline</td>
</tr>
<tr>
<td>$s_1$</td>
<td>1.25</td>
<td>Swell factor after blasting the overburden</td>
</tr>
<tr>
<td>$s_2$</td>
<td>1.30</td>
<td>Swell factor after dumping material to the spoil pile</td>
</tr>
<tr>
<td>$t_d$</td>
<td>15 sec</td>
<td>Time to fill the dragline bucket</td>
</tr>
</tbody>
</table>

Table 1: Parameter values

Variables

We use the following variables to describe the system.

Let $N$ be the total number of $O$ sub-blocks.
There are $N$ corresponding $D$ sub-blocks.

An $O$ sub-block is represented by $j$.
   $j = 1, \ldots, M$ are the $O$ sub-blocks in the highwall bench blocks.
   $j = M + 1, \ldots, P$ are the $O$ sub-blocks in the overburden block.
   $j = P + 1, \ldots, N$ are the $O$ sub-blocks in the bridge.

$V_j$ is the volume of the $j$th $O$ sub-block.

$d_j$ is the distance from dragline position $j$ to position $(j + 1)$.

The digging efficiency is $e = 1$ when digging below the operating level and $e = 0.7$ when digging above the operating level.

For a vertical distance $h$ metres, the hoist and lowering times are given by the dragline
For a horizontal swing angle $\theta$ degrees, the swing forward and swing back times are given by

\[
\text{swing forward time} \quad t_f = 0.1188\theta + 7.4764 \text{ sec} \\
\text{swing back time} \quad t_b = 0.1177\theta + 6.4036 \text{ sec}
\]
The time needed to excavate the $j$th O sub-block, and hence the $j$th O-D pair, is given by

$$T_j = \frac{s_1V_j}{V_b}(t_d/e + t_{ob})$$

(8)

where

\[ e = \begin{cases} 
0.7, & j = 1 \ldots M \\
1.0, & j = M + 1 \ldots N 
\end{cases}, \quad s_1 = \begin{cases} 
1.25, & j = 1 \ldots P \\
1.00, & j = P + 1 \ldots N 
\end{cases} \]

and $V_j/V_b$ is the number of cycles of the "fill, hoist, swing forward, swing back, lower" bucket movement sequence for volume $V_j$.

**Total excavation time**

The total excavation time $T$ is

$$T = \sum_{j=1}^{N} T_j + \sum_{j=1}^{N} \frac{d_j}{v}$$

(9)

Note that at this stage we still have to set up a method for calculating $h_j$, $\theta$ and $d_j$. We can do this by choosing a suitable set of axes and then linking the centres of the O and D sub-blocks via an arbitrary (dragline) position on the operating bench.

During this process we need to consider a way of stating the combinatorial problem arising from the mapping between the O and D sub-blocks that will allow us to minimise the value of $T$.

**Constraints**

There are constraints associated with the sequencing of the O-D pairs, with the dragline dimensions and with the volumes of overburden excavated.

When setting up the sequence of O-D mapped pairs we need to consider the following points:

- The horizontal distance between O sub-blocks and possible D sub-blocks cannot exceed twice the dragline reach.

- When excavating the overburden and bridge we cannot excavate an O sub-block until the one above it has been removed.

- When building the spoil pile we cannot fill a D sub-block until the one below it has been completed.

These points relate obviously to the general digging sequence in which, if the dragline starts in position 1 as in figure 1 and moves out onto the bridge, the spoil dump locations
move further out from the (old) highwall.

The second set of constraints relates to the strip geometry and the dragline geometry. To write these constraints we need to refer to figure 2.

![Figure 2: Dragline Range Diagram](image)

We have a volume constraint which, assuming that the volume of material used to build the new bridge is equal to the volume removed in excavating the old bridge, we can write as

$$s_2(V_H + V_O) = V_s$$

(10)

where $V_H$ is the volume of the highwall bench, $V_O$ is the volume of the overburden, $V_s$ is the volume of the spoil pile and $s_2$ is the swell factor relating the prime volumes to the volume in the spoil pile.

The remaining constraints relate to the dragline dimensions:

- The horizontal distance from 5 metres before the edge of the bridge to the spoil pile peak cannot exceed the dragline reach. The 5 metres is a safety factor, the dragline cannot be moved any closer to the edge of the bridge than this distance.

- The vertical distance from the operating level to the spoil pile peak cannot exceed the dragline maximum dump height.
• The vertical distance from the operating level to the top of the highwall bench block cannot exceed typically 30 metres.

• The vertical distance from the operating level to the bottom of the coal seam cannot exceed the dragline maximum digging depth. Here we are assuming that the coal seam is horizontal.

The formulation of these constraints is straightforward once we have chosen a co-
ordinate system to suitably describe the dragline and sub-block locations. They are given as equations 14 and 15.

4. Excavating sub-blocks

Figure 3 shows the division of the overburden material into typical sub-blocks. The $O$ sub-blocks are shown in figure 3 (a) and the $D$ sub-blocks in figure 3 (b). A view from above is given in figure 4.

The blocks are excavated in order from 1 to 4. The origin of the $j$th sub-block is given by $O_j$ which is taken as the centre of mass of the origin sub-block. The corresponding destination is given by $D_j$. The locations of the $D_j$ are the points at which the material is released so that the spoil pile sub-blocks will build naturally at the repose angle $\beta$. The location of the dragline for the $j$th $O-D$ pair is given by $P_j$.

Note that, for demonstration purposes, we have assumed that the bridge is built from the keycut (region 1) and from the material above the operating level (region 2). We also assume that region 2 is excavated from two dragline positions. The first position is the same as that for the keycut, in this position we have only to move spoil material to the edge of the highwall. The rest of region 2 must be excavated from a new position so that we can reach position $D_{2b}$ at the edge of the bridge.

Referring to figure 4, we can see that there is no point in moving the dragline position $P_j$ closer to the destination location $D_j$ than the length given by the effective dragline reach, since this will only increase the swing angle for a given $O-D$ pair.

This means that we can fix the position of $P_j$, in the direction perpendicular to the strip, from the location of the corresponding destination $D_j$. Further, we can calculate the location of a particular destination from the geometry shown in figures 3 and 4, together with the parameter values given in Table 1.
Figure 3: Origin and destination sub-blocks

Figure 4: The layout of figure 3 shown from above
5. Simulating dragline operation

Given a set of parameters as in Table 1, the O-D geometry shown in figures 3 and 4, and equations 5 to 9, we can develop a simulation model of the dragline operation. We need to specify in addition, the width at the bottom of region 1 (the keycut) and the width of region 3.

Using this data we can calculate the origin positions, the size of the bridge and the locations of the sub-block destination positions. We can then calculate the excavation times needed for the different O-D pairs.

It is possible to specify the system completely except for the dragline position coordinates in the direction along the strip. Consequently, a useful simulation model can be developed in which a user could study a series of 'what if' scenarios in which the dragline positions can be specified with a single variable.

Such a simulation model can be made more accurate by dividing the overburden into smaller regions.

At present a prototype model, in which the regions 1–4 have each been divided into three horizontal layers, is under development. This prototype model is giving realistic output results at the current stage of verification.

6. Optimising overall productivity

In this section we develop formulae to find the optimum operating level $y_b$ and pit width $X_{pit}$ for the dragline. We use figure 5 as reference.

![Dragline operation geometry](image)

Figure 5: Dragline operation geometry
From figure 5 we have \[ Y_s = 0.5 X_{pit} \tan \beta \] and substituting for \( Y_s \) in the volume balance equation: (expanded overburden volume) = (spoil pile volume) which can be written

\[ s_2 X_{pit} Y_{ob} = X_{pit}(y_p - Y_s) + \frac{1}{2} X_{pit} Y_s \]

gives

\[ y_p = s_2 Y_{ob} + \frac{1}{4} X_{pit} \tan \beta \]  \hspace{1cm} (11)

then from \( x_p = X_{pit} + y_p/\tan \beta \) we have

\[ x_p = 1.25X_{pit} + \frac{s_2 Y_{ob}}{\tan \beta} \]  \hspace{1cm} (12)

**The co-ordinates \((x_b, y_b)\) of the top right corner of the bridge**

The values of \( x_p \) and \( y_p \) depend on the dragline dimensions and the location \((x_p, y_p)\) of the spoil pile peak. We have from \( x_b = x_p - D_{reach} \) (where \( D_{reach} \) includes a safety factor representing the distance that the dragline must remain from the edge of the bridge)

\[ x_b = 1.25X_{pit} + \frac{s_2 Y_{ob}}{\tan \beta} - D_{reach} \]  \hspace{1cm} (13)

Note that if \( x_b \leq x_d = X_{pit} - \frac{Y_b}{\tan \alpha} \) then we do not need a bridge.

The operating level \( y_b \) is one of the variables we wish to find, it is constrained by the dragline dimensions so that:

\[ y_p - D_{up} \leq y_b \leq D_{down} \]  \hspace{1cm} (14)
\[ Y_{coal} \leq y_b \leq Y_{coal} + Y_{ob} \]  \hspace{1cm} (15)

**The co-ordinates \((x_c, y_c)\) of the outer corner of the bridge**

From \( y_c = (x_c - X_{pit}) \tan \beta \) and \( (x_c - x_b) \tan \beta = (y_b - y_c) \):

\[ x_c = \frac{1}{2}(x_b + X_{pit} + \frac{y_b}{\tan \beta}) \]  \hspace{1cm} (16)

\[ y_c = \frac{1}{2}(y_b + (x_b - X_{pit}) \tan \beta) \]  \hspace{1cm} (17)
The effect of operating level and bridge area on excavation time

For the particular model under consideration, the total excavation time will be influenced by:

a) the cross sectional area $A_b$ of the bridge, if one is built,

b) the fact that digging efficiency is reduced to $e = 70\%$ when removing overburden from above the operating level $y_b$.

Assuming that for an efficient digging operation, excavation times will depend on the volumes (and here, on cross sectional areas) of material shifted, we can write an equation for the excavation time $T$. The equation for $T$ will contain a term related to excavating the bridge and a term related to operating below the surface level.

For purposes of comparison we are interested in a normalised measure of excavation time. We will use $T/X_{pit}$, the excavation time per unit (pit) width. This is proportional to the excavation time to uncover a fixed amount of coal and hence maximum productivity is obtained by minimising $T/X_{pit}$.

From

$$ T \propto A_b + X_{pit} \left\{ (y_b - Y_{coal}) + \frac{1}{e} (Y_{coal} + Y_{ob} - y_b) \right\} $$

we have

$$ \frac{T}{X_{pit}} \propto \frac{A_b}{X_{pit}} + (y_b - Y_{coal}) + \frac{1}{e} (Y_{coal} + Y_{ob} - y_b) $$

where the bridge area $A_b$ is given by

$$ A_b = y_b (x_c - x_d) - \frac{y_b^2}{2 \tan \alpha} - \frac{1}{2} (x_c - x_b)^2 \tan \beta - \frac{1}{2} (x_c - X_{pit})^2 \tan \beta $$

and our aim is to minimise the right hand side of equation 18 with respect to $X_{pit}$ and $y_b$.

The effect of operating level on excavation productivity

Figure 6 shows the relationship between excavation productivity and operating level for a number of different pit widths.

The curves for $X_{pit} = 60-100$ represent cases where we need to build a bridge. For the curve $X_{pit} = 20$ no bridge is needed. We will look at this factor in more detail in the next section.

The diagram shows that, for a given pit width it is, in general, possible to find an operating level that maximises the productivity (minimises the excavation time per unit width) and that the optimal productivity is greater for larger pit widths.
We find the value $y_b^*$ for which this optimum occurs by using (11)-(17) to write the equation for $T/X_{pit}$ in terms of $y_b, X_{pit}$ and known parameters. Then, differentiating (18) with respect to $y_b$ gives:

$$\frac{\partial (T/X_{pit})}{\partial y_b} = \frac{1}{8} + \left[ 1 - \frac{1}{e} \right] + \frac{1}{X_{pit}} \left[ \frac{s_2Y_{ob}}{2 \tan \beta} - \frac{1}{2} D_{reach} + \frac{y_b}{\tan \alpha} + \frac{y_b}{2 \tan \beta} \right]$$

(19)

and

$$\frac{\partial^2 (T/X_{pit})}{\partial y_b^2} = \frac{1}{X_{pit}} \left[ \frac{1}{\tan \alpha} + \frac{1}{2 \tan \beta} \right] > 0 \text{ so we have a minimum.}$$

Setting the RHS of (19) to zero and writing

$$k_1 = D_{reach} \frac{s_2Y_{ob}}{\tan \beta}, \quad k_2 = \frac{1}{\tan \alpha} + \frac{1}{2 \tan \beta}, \quad k_3 = \frac{1}{8} + (1 - \frac{1}{e})$$

gives

$$y_b^* = \frac{-1}{k_1} (k_3X_{pit} - \frac{k_2}{2})$$

(20)
The effect of pit width on excavation productivity

Figure 7 shows the relationship between excavation productivity and pit width for a number of different operating levels.

\[ T/X_{pit} \] (arbitrary scale)

![Graph showing the relationship between excavation productivity and pit width for different values of \( y_b \). The graph includes curves for \( y_b = 50, 40, 25, 15 \).]

For fixed values of \( y_b \), (18) shows some interesting properties.

If \( y_b > k_1(1 + \sqrt{1 + 2k_2 \tan \beta})/2k_2 \), which we calculate below and which corresponds to \( x_b > x_d \), then the function behaves as shown by the curves for \( y_b = 25-50 \). Physically these values of \( y_b \) correspond to situations where we need to build a bridge in order to reach the horizontal peak of the spoil pile. The productivity increases with pit width but with a law of diminishing return as the width increases. We should use the maximum pit width possible.

If \( y_b \) is less than the value given in the previous paragraph, then the function behaves as shown by the curve for \( y_b = 15 \). In this case (18) is not physically relevant. It gives a solution corresponding to a situation in which no bridge is needed and an unnecessary volume of material is simply moved along the pit.

Differentiating (18) with respect to \( X_{pit} \) gives

\[
\frac{\partial(T/X_{pit})}{\partial X_{pit}} = \frac{1}{2X_{pit}^2} \left[ (k_1y_b - k_2y_b^2) + \frac{1}{2} \tan \beta (k_1^2 - (X_{pit}/4)^2) \right]
\]

(21)

with

\[
\frac{\partial^2(T/X_{pit})}{\partial X_{pit}^2} = -\frac{1}{X_{pit}^3} \left[ (k_1y_b - k_2y_b^2) + \frac{1}{2} k_1^2 \tan \beta \right]
\]

(22)

The second derivative switches from negative to positive, corresponding to the change
in behaviour shown by the difference between the curves for \( y_b = 15 \) and \( y_b = 25 \) in figure 7, when

\[
(k_1 y_b - k_2 y_b^2) + \frac{1}{2} k_1^{2} \tan \beta = 0 \quad \Rightarrow \quad y_b = \frac{k_1 (1 \pm \sqrt{1 + 2k_2 \tan \beta})}{2k_2}
\]

We require \( y_b > 0 \) which gives

\[
y_b = \frac{k_1 (1 + \sqrt{1 + 2k_2 \tan \beta})}{2k_2}
\]  \hspace{1cm} (23)

For this value of \( y_b \) the outer top of the bridge coincides with the edge of the highwall \((i.e. \ x_b = x_d)\).

For \( y_b \) greater than this value the second derivative will be positive and we obtain the behaviour shown by the curves \( y_b = 25 - 50 \) in figure 7.

If \( y_b \) is less than this value then (18) is not physically relevant since we do not need a bridge. We remove the area term \( A_b \) from (18) and note, obviously, that the productivity increases as we increase \( y_b \) and reduce the amount of material removed from above the operating level.

**Optimising productivity using non linear programming**

The above results were confirmed by using the MINOS Non Linear Programming optimiser to minimise (18) subject to the constraints of (14, 15) and with the lower bound on \( y_b \) set by (23). The program was run with different upper bounds (but all less than 90 metres) on \( X_{pit} \).

In all cases the optimisation drove the pit width to its upper bound and returned an optimum value for \( y_b \) as given by substituting the upper bound of the pit width into (20).

**Conclusions**

The above discussion assumes that we will not triple handle the overburden material. This means that we are restricting the maximum pit width to be less than the maximum effective reach of the dragline.

In this situation the results indicate that the most effective operation is obtained by choosing the maximum pit width possible and then calculating the operating level using

\[
y_b^* = -\frac{1}{k_1} (k_3 X_{pit} - \frac{k_2}{2})
\]
7. Solving the combinatorial part of the problem with Dynamic Programming

If we know the excavation rate as a function of location across the overburden block then we can use Dynamic Programming to determine an optimal way of dividing the overburden into sub-blocks corresponding to the different dragline excavation positions.

The DP procedure is applied in the context of the geometry shown in figure 8. We divide the overburden into \(k\) excavation positions \(P_1, P_2, \ldots, P_k\). With reference to figure 3 on page 9, once we have specified the width of the base of the keycut and the total width of region 3 (which we may divide into sub-blocks as here), the first and last positions are given by the overall geometry.

Let \(s_n\) be the distance to the origin (bottom left corner) of block \(n-1\), then

\[
s_n = \sum_{i=1}^{n-1} y_i, \quad n = 2, 3, \ldots, k
\]

with \(s_1 = 0\).

Also, let \(f_n(s_n)\) denote the optimal (minimal) time required to remove the remaining overburden, given that after removing \(n - 1\) blocks we are in position \(s_n\). Then \(f_{k+1}(s_{k+1}) = 0\), and the optimal time to move the entire cut is given by \(f_1(0)\).

We can show that the following functional equation holds.

\[
f_n(s_n) = \min_{y_n \in D(s_n)} \{T(s_n, y_n) + f_{n+1}(s_n + y_n)\}, \quad 1 \leq n \leq k, \quad s_n \in S_n
\]

where

- \(D(s_n)\) = the set of feasible values of \(y_n\) given that we are in:
  - \(S_n\) = the set of (discrete) feasible values of \(s_n\).
  - \(T(s_n, y_n)\) = the time to move the \(n\)th sub-block to the spoil pile.

(25) can be solved rapidly — assuming that the function \(T\) can be evaluated rapidly. This formulation takes advantage of the fact that the model is “separable” in a dynamic programming sense (Sniedovich, 1992).
8. Sequencing the excavation from the keycut and highwall blocks

In this section we consider that part of the excavation where the dragline operator can choose to dig from either the highwall bench block or from the keycut region. In particular, we investigate whether there is an optimal sequencing of the digging from these two regions that will minimise the excavation time.

By making appropriate simplifications we can remove excess complexity from the problem but retain its essential elements. Here, we look at a simplification of the operations in which we reduce the number of spatial dimensions used to describe the overburden. We assume average behaviour for two dimensions and then conduct the analysis with respect to the remaining dimension.

We retain the single dimension across the pit and assume that, for each section of the overburden (averaged down and along the pit), the times to fill, hoist and lower the bucket remain relatively constant. We take the major variable part of the total excavation time to be the swing times.

**Problem formulation**

Figure 9 shows the simplified geometry for the sequencing problem. Side ‘a’ is taken to represent the overhand block and side ‘b’ represents the keycut.

![Diagram showing a and b sides of the excavation](image)

Figure 9: Normalised diagram for digging from both sides

The distances from the dragline path to the centre lines of the overburden blocks can be normalised to 1 without loss of generality.
We use the following notation in the formulation of the problem.

**Parameters**

\[ a_1, a_2 \quad \text{the end co-ordinates of side 'a'} \]
\[ b_1, b_2 \quad \text{the end co-ordinates of side 'b'} \]
\[ V_a \quad \text{the volume of overburden in side 'a'} \]
\[ V_b \quad \text{the volume of overburden in side 'b'} \]
\[ e_a \quad \text{the bucket filling efficiency when excavating from side 'a'} \]
\[ e_b \quad \text{the bucket filling efficiency when excavating from side 'b'} \]
\[ A \quad \text{the distance moved by the dragline in excavating all of side 'a'} \]
\[ B \quad \text{the distance moved by the dragline in excavating all of side 'b'} \]
\[ C \quad \text{the distance moved by the dragline after completely excavating the overburden to the spoil pile} \]

We assume that the volumes have been corrected to allow for swelling arising from blasting the overburden prior to excavation and from dumping the overburden to the spoil pile.

In practice, bucket filling is less efficient when digging the overhand block because the excavation occurs at or above the dragline operating level and it is difficult to fill the bucket. We designate digging efficiencies \( 0 \leq e_a \leq 1 \) for side 'a' and \( 0 \leq e_b \leq 1 \) for side 'b' to allow for this fact. A digging efficiency of \( e_a = 0.8 \) indicates that the bucket is filled to 80% of its capacity when digging from above the operating bench on side 'a'.

**Variables**

\[ p \quad \text{the proportion of the total overburden that has been excavated so far in the current cycle} \]
\[ \eta(p) \quad \text{the proportion of side 'a' excavated when a proportion } p \text{ of the total overburden has been excavated} \]
\[ \xi(p) \quad \text{the proportion of side 'b' excavated when a proportion } p \text{ of the total overburden has been excavated} \]
\[ \theta \quad \text{the swing angle on Side 'a'.} \]
\[ \phi \quad \text{the swing angle on Side 'b'.} \]

For convenience, we ignore the 90° component and take the swing angles \( \theta \) and \( \phi \) to be as shown in figure 9.

We initially locate the dragline at position (0, 0) as shown in figure 9. The overhand overburden block is centred on the line \((a_1, 1)\) to \((a_2, 1)\). The keycut overburden block is centred on the line \((b_1, -1)\) to \((b_2, -1)\). The digging direction is constrained to be from \((a_1, 1)\) to \((a_2, 1)\) and from \((b_1, -1)\) to \((b_2, -1)\).
We assume that the dragline position is always a fixed distance from the spoil pile dumping point. This distance is set as the effective dragline reach so as to minimise the angle through which the dragline must turn. As the overburden is excavated the dumping point shifts outwards and the dragline moves accordingly.

The objective function

Suppose that the proportion of overburden removed at some stage is \( p \) and that we increase this to \((p + \Delta p)\) by increasing the proportion excavated from side \('a'\) from \( \eta \) to \( (\eta + \Delta \eta) \) and the proportion excavated from side \('b'\) from \( \xi \) to \( (\xi + \Delta \xi) \).

At this stage, the dragline is at position \( P_1 \) shown in figure 9. The excavation point for side \('a'\) is at \( P_2 \) and the excavation point for side \('b'\) is at \( P_3 \).

The volume of material removed from side \('a'\) is \( V_a \Delta \eta \) and, assuming a unit bucket size, the number of bucket loads involved in this excavation is \( (V_a/e_a)\Delta \eta \).

The angle turned through by the dragline in carrying out this excavation is then given by

\[
\frac{V_a}{e_a} \Delta \eta \theta
\]  

(26)

Similarly, if we excavate a volume \( V_b \Delta \xi \) from side \('b'\) then the angle turned through by the dragline is

\[
\frac{V_b}{e_b} \Delta \xi \phi
\]  

(27)

The angle turned through in increasing the proportion of overburden excavated from \( p \) to \((p + \Delta p)\) is therefore

\[
\frac{V_a}{e_a} \Delta \eta \theta + \frac{V_b}{e_b} \Delta \xi \phi
\]  

(28)

The angles and digging constraints associated with the problem are most conveniently expressed in terms of the positions of the dragline and excavation points on sides \('a'\) and \('b'\). We therefore rewrite (28) in terms of the distances associated with these positions.

For the simplified case under consideration, we specify averaged behaviour for the dimension across the strip. We assume that the dumping position is at the outer edge of the spoil material and thus depends on the proportion of total overburden excavated. The dragline moves so that it is always a fixed position from the dump position. When a proportion \( p \) of the overburden has been excavated the distance moved by the dragline is thus \( Cp \). This distance is made up of a displacement \( A \eta \) from digging side \('a'\) and a displacement \( B \xi \) from digging side \('b'\). At the completion of digging we have
\[ A + B = C \]  

(29)

We let \( A = kV_a \) and \( B = kV_b \), where \( k \) depends on the physical dimensions of the strip. The angle turned through in excavating a proportion \( \Delta p \) of the total overburden can thus be written as

\[
\frac{1}{k} \left( \frac{A}{e_a} \frac{\Delta \eta}{\Delta p} \theta + \frac{B}{e_b} \frac{\Delta \xi}{\Delta p} \phi \right) \Delta p
\]  

(30)

and the total angle turned through by the dragline as

\[
\frac{1}{k} \int_0^1 \left( \frac{A}{e_a} \eta' \theta + \frac{B}{e_b} \xi' \phi \right) dp
\]  

(31)

So our problem is:

\[
\text{minimise} \quad \int_0^1 \left( \frac{A}{e_a} \eta' \tan^{-1} \mu + \frac{B}{e_b} \xi' \tan^{-1} \nu \right) dp
\]  

(32)

where the equations for \( \mu \) and \( \nu \) are given as the first two constraints below.

**Constraints**

From figure 9 we have that

\[ \mu = (Cp - a_1) - (a_2 - a_1) \eta \]  

(33)

\[ \nu = (Cp - b_1) - (b_2 - b_1) \xi \]  

(34)

We also require

\[
\eta(0) = 0, \quad \eta(1) = 1 \quad \xi(0) = 0, \quad \xi(1) = 1
\]  

(35)

In addition we have

\[ A\eta + B\xi = Cp \]  

(36)

and differentiating (36) gives

\[ A\eta' + B\xi' = C \]  

(37)

Finally, the dragline digs only from overburden to spoil so \( \eta' \geq 0 \) and \( \xi' \geq 0 \), and from (29) and (37) we have

\[
0 \leq \eta' \leq \frac{A+B}{A} \quad 0 \leq \xi' \leq \frac{A+B}{B}
\]  

(38)

The problem given by (32)–(38) can be solved using either an optimal control approach or calculus of variations with Lagrange multipliers. Initial work indicates that the op-
Optimal solution has bang-bang control, switching between \( \eta' = 0 \), \( \xi' = (A + B)/B \) and \( \eta' = (A + B)/A \), \( \xi' = 0 \).

Using a discrete problem formulation, preliminary numerical analyses of the optimal schedules for digging from the two sides gave only two general excavation schedules for \( \eta' \) and \( \xi' \):

- Excavate all of side ‘a’ (\( \xi' = 0 \)), then excavate side ‘b’.
- Excavate all of side ‘b’ (\( \eta' = 0 \)), then excavate side ‘a’.

Which side is excavated first depends on the geometry of the problem. If the problem is symmetric both schedules are optimal.

A Dynamic Programming approach

Dynamic programming provides an alternative way of solving specific cases of this optimisation problem, and being a numerical technique will conveniently allow a more detailed description of the digging time.

The state of the problem after a proportion \( p \) of overburden has been excavated is given by \( \eta(p) \) so a two dimensional table with state \( \eta(p) \) and stage \( p \) can be used to implement the dynamic programming. As we have seen above, the solution is either \( \eta' = 0 \) or \( \xi' = 0 \) at each small step so the dynamic programming needs to examine only these two options at each table entry. Note that multiple local optima could exist.

We can calculate average angles through which the dragline must swing to change state by moving from \( p \) to \( (p + p/n) \), with \( n \) representing the number of steps in the analysis. These angles are calculated for digging from both the ‘a’ and ‘b’ sides.

It is then possible to calculate the minimum total angle through which the dragline must turn in order to make the transition from any given state to the final condition \( \eta(1) = 1 \) by using the equation

\[
g(s, p) = \min\{g(s + 1, p + 1) + \theta_s , g(s, p + 1) + \phi_s\}
\]

In (39), \( g(s, p) \) represents the minimum total angle to move from state \( s \) stage \( p \) to the final condition \( \eta(1) = 1 \). \( \theta_s \) is the angle when digging from side ‘a’ and \( \phi_s \) is the angle when digging from side ‘b’.

Calculations of \( g(s, p) \) for different cases give results consistent with those noted above, namely that the optimal digging schedule is to either dig all of side ‘a’ first then side ‘b’, or vice versa depending on the particular geometry and excavation rates.
9. Conclusions

The Study Group identified a number of possible approaches to tackling the problem of optimising dragline operations. The techniques discussed in this report provide the first steps toward developing full 3-dimensional optimisation methods for the problem. Initial numerical work based on these methods has provided some insight into the parameters which are important in improving the dragline operating efficiency. A simulation model of the operation has been used for a preliminary investigation of the effects of different methods of walking the dragline across the operating bench.

The methods considered in the report are for the particular method of removing overburden outlined in the introduction. There are however a number of quite different operational techniques that can be employed when using a dragline to remove overburden.

Of particular importance is the blasting technique used to loosen the overburden. The charges can be set to produce a number of different possible profiles of the blasted overburden. These profiles determine the subsequent possible movements of the dragline machines.

The problem thus involves not just the optimisation of a given set of dragline operational techniques but a comparison of different operational techniques used in conjunction with the initial blasting patterns chosen for a particular type of overburden.

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References


Optimised dragline planning model


