ATOMISATION OF MOLTEN ZINC

Zinc dust is produced by an atomisation process in which air at 17 atmospheres pressure is directed through a narrow annular nozzle surrounding a stream of molten zinc. Industrially, it is important to control the size distribution of zinc particles. It is therefore necessary to understand the features of the process, guided where appropriate by simple mathematical models and estimates. This report examines qualitative features using simple models for number of drops produced, estimates for air speed and swirl, heat transfer in the molten zinc and droplets, and droplet collisions and stability. Results from a literature survey are presented. At this stage, the construction of more elaborate mathematical models for the process is not warranted; rather, we expect that better understanding of the process requires an experimental program involving flow visualisation.

1. Introduction

Pasminco Australia Ltd operates several zinc smelters in Australia. A key stage in the production of zinc is the purification of a zinc salt solution by the addition of fine zinc dust. This causes a displacement and cementation reaction in which more electropositive impuritic metals are displaced by metallic zinc and removed by filtration. The solution is subsequently electrolysed to produce zinc metal. The importance of zinc dust to the process is evident in that some 5% of the total zinc production is diverted to the atomisation plant.

The Mathematics-in-Industry Study Group was asked to examine the atomisation of molten zinc in order to understand the process better and, hopefully, to suggest ways of producing particle distributions with advantageous properties.

The zinc atomisation process is sketched in figure 1. The atomisation equipment was designed and built by Richard Snow and Mark Geeves of Pasminco. Molten zinc at about 600°C flows vertically under a pressure head of about 600 mm of molten zinc down a tube of diameter 3 mm bored into an axially symmetrical steel nozzle. The nozzle protrudes about 9 mm beyond the flat bottom of a steel chamber containing air at about 17 atmospheres pressure. The air is directed in a high speed annular jet around the molten zinc through an annular nozzle of area \(160 \times 10^{-6} \, \text{m}^2\). Although it is not evident in figure 1, the air flow has a substantial additional complication due to swirl induced by the inlet geometry of the air chamber.

The molten zinc is atomised rapidly into zinc dust which is collected and partially sorted for size in a series of cyclones. Approximately \(10^3\) kg of zinc dust is manufactured.
per hour. Detailed observation of the process is difficult because of the hot, dusty and noisy environment.

The zinc dust commonly produced by Pasminco has the typical particle size distribution shown in figure 2. Pasminco would like to be able to produce zinc dust with a relatively narrow size distribution. Zinc dust with an increased amount of small particles increases the cadmium impurities of the purified solution, whilst large particles increase the cobalt impurities. A size distribution with minimal small and minimal large particles will work efficiently on both these impurities.

Pasminco has carried out tests using various nozzle geometries, air pressures, zinc temperatures and zinc additives. They also have practical operating experience of the atomisation plant. They do not have, however, a detailed understanding of the various fluid processes (air flow, drop formation mechanism) that take place during atomisation.

Atomisation is, of course, a very common process. Other applications include fuel injection in engines, spray painting, and production of powdered milk and other foodstuffs (see e.g. Lefebre, 1989, for further details). Computer searches of standard engineering and science databases identified 14 references jointly mentioning ‘atomiz’ or ‘atomis’ with ‘liquid metal’ and ‘zinc’. Of these references, only Smith et al. (1985) describe a similar method to that of Pasminco. Several research groups (e.g. Dunkley & Palmer, 1986) use water and not air as the working fluid; others (e.g. Kruus, 1988) describe a process involving ultrasound, whilst others (e.g. Halada et al., 1990) describe centrifugal atomisation.

A large number of other references on drop formation and stability were also identified by various means including follow-up library searches. Most of these references were concerned with scales that are inappropriate for the atomisation process; details have nevertheless been forwarded to Pasminco.
Work at the Study Group was mainly concerned with establishing the main features of the air flow (Section 2), heat transfer for the molten zinc stream and for droplets (Section 3), and droplet stability and collisions (Section 4). Taken together, these considerations and their simple estimates enabled a broad understanding of the atomisation process. There was, however, no point at this stage in proceeding to detailed mathematical models; such models require extensive time for development, and need to be based on a better experimental understanding than available at present for the process.

The main conclusions of the report are presented in Section 5. To summarise them: the air flow is supersonic after it leaves the nozzle; swirl is a significant feature of the flow; the droplets take sufficiently long to solidify and air speeds are such that collisions between droplets are inevitable; observations on the effects of various nozzles can be explained to a certain extent; and detailed mathematical/computational work on this problem should not be undertaken lightly given that the flow is three-dimensional, involves three phases, is supersonic and subsonic in various regions, and involves free boundaries. Flow visualisation and physical experimentation is recommended.
2. Features of the air flow

We begin with an estimate of the swirling velocity. Figure 3 illustrates the geometry of the air chamber and nozzle as seen from directly below. Suppose that air at density $\rho_1$ and pressure $p_1$ is introduced with speed $v_1$ into the circular chamber through a tube with cross-sectional area $A_1$. This tube is offset a mean distance $R_1$ from the centre of the chamber. Suppose also that the air leaves the chamber through the annular nozzle of area $A_2$ at pressure $p_2$, density $\rho_2$ and with axial speed $v_2$ and swirl speed $w_2$. The annular nozzle has a mean radius $R_2$.

![Figure 3: The air chamber and nozzle as seen from below.](image)

Conservation of mass requires that

$$\rho_1 v_1 A_1 = \rho_2 v_2 A_2$$

If wall friction and other viscous effects of the high speed air flow inside the chamber are neglected, the nett angular momentum of the air flow inside the chamber will be constant. This gives

$$\rho_1 v_1^2 A_1 R_1 = \rho_2 v_2 w_2 A_2 R_2$$

from which it follows that

$$v_1 R_1 = w_2 R_2$$

The design values are $R_1 = 0.05$ m and $R_2 = 0.0095$ m, from which we have $w_2 = 5.3 v_1$.

The value $v_1$ comes from mass flow considerations. The volume flux of air (at room temperature and pressure) per hour is 1000 m$^3$. Since the value $v_1$ is measured at 17 atmospheres and room temperature, we have

$$A_1 v_1 = \frac{1000}{17 \times 3600} \text{ m}^2\text{s}^{-1}$$
or \( v_1 = 58 \text{ ms}^{-1} \) since \( A_1 = \pi(9.5 \times 10^{-3})^2 \text{ m}^2 \). The swirling speed \( w_2 \) is therefore approximately \( 300 \text{ ms}^{-1} \), a value near the speed of sound. In general, supersonic and transonic flow have the attendant possibility of shocks, although this is not likely to be a feature of the swirling component of this particular flow.

Figure 4: Compressible flow in a channel of varying cross-section.

We now consider compressible flow of air in a channel of varying cross-section (see figure 4). This is a widely studied topic in gas dynamics. A treatment with an engineering focus on flow measurement is given, for example, by Ower & Pankhurst (1977, chapter 7), and our treatment below is an extension of section 1.64 of Milne-Thompson's (1968) book.

Consider a swirling flow in the \( x \)-direction in a duct of varying cross-section \( A \) as shown in figure 4. Let the density, axial speed, swirl speed and pressure be \( \rho, v, w, p \) respectively. Let the flow be adiabatic with \( p = K\rho^\gamma (\gamma = 1.4) \), and \( c^2 = \frac{dp}{d\rho} = \frac{\gamma p}{\rho} \) where \( c \) is the speed of sound. Conservation of mass gives

\[
A \rho v = Q = \text{constant} \quad (2)
\]

whilst Bernoulli's law gives

\[
\frac{1}{2} (v^2 + w^2) + \frac{\gamma}{\gamma - 1} \frac{p}{\rho} = \text{constant}
\]

or

\[
\frac{1}{2} (v^2 + w^2) + \frac{\gamma}{\gamma - 1} K\rho^{\gamma - 1} = \text{constant} \quad (3)
\]

Assume also that the swirling flow can be represented as a rotating disk with peripheral speed \( w \). Conservation of angular momentum then gives

\[
\frac{1}{2} \rho A \frac{w}{\pi \sqrt{A/\pi}} = \text{constant}
\]
or

\[ \rho A^{1/2} w = \text{constant} \]  \hspace{1cm} (4)

Differentiation of (2-4) with respect to \( x \) yields

\[ \frac{1}{A} \frac{dA}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{v} \frac{dv}{dx} = 0 \]

\[ \frac{v}{A} \frac{dv}{dx} + w \frac{dw}{dx} + \frac{c^2}{\rho} \frac{d\rho}{dx} = 0 \]

\[ \frac{1}{2A} \frac{dA}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{w} \frac{dw}{dx} = 0 \]

from which the terms involving \( dp/dx \) and \( dw/\rho dx \) can be eliminated to obtain

\[ (v^2 + w^2 - c^2) \frac{1}{v} \frac{dv}{dx} = (c^2 - \frac{1}{2} w^2) \frac{1}{A} \frac{dA}{dx} \]  \hspace{1cm} (5)

Just upstream of the outlet nozzle, \( dA/dx < 0, c^2 - w^2/2 > 0 \) and \( u^2 + w^2 - c^2 > 0 \) (refer to the above estimate for the swirl velocity \( w \)). Hence it follows that \( dv/\rho dx < 0 \) just upstream of the nozzle. At the narrowest point of the nozzle, \( dA/dx = 0 \) and either \( v^2 + w^2 = c^2 \) (impossible in view of the estimate for the swirl speed) or \( dv/\rho dx = 0 \). Downstream of the exit nozzle, similar arguments show that \( dv/\rho dx > 0 \). That is, the axial flow speed decreases before the exit point, reaches a minimum and then increases. In general and in the absence of swirl, the minimum speed reached in the nozzle is the speed of sound – the speed at which pressure fluctuations cannot be propagated upstream. The nozzle is then choked. In our case, the minimum speed reached is affected by the amount of swirl and would be given by integration of (5) along the axial direction.

This concept of flow through the nozzle requires some modifications in order to give a better representation of reality.

- There will be boundary layers adjacent to the walls of the nozzle, and these boundary layers will probably become turbulent.
- Downstream of the nozzle, the air flow expands and becomes supersonic. Shocks will form where high speed axial flow is incident on parts of the nozzle.
- Where the flow expands past a sharp corner, Prandtl-Meyer expansion fans will occur.

A schematic representation of the flow can be constructed using the flow visualisation pictures compiled by Van Dyke (1982). Our understanding of the flow is presented in figure 5. This schematic representation needs to be corroborated by flow visualisation work.
An indirect inference on the supersonic nature of the flow can be made by examining
the nature of the very loud sound (130 dbA) that accompanies the zinc dust manufactur-
ing process. If the sound has a rather sharp peak in the frequency spectrum, this would
be indicative of vortex shedding at regular intervals behind the steel nozzle. That is, it
would indicate subsonic flow. On the other hand, supersonic flow with turbulent bound-
dary layers – as we believe to be the case – would be characterised by a more uniform
sound spectrum.

\[
\pi(1.5 \times 10^{-3})^2 6430 \bar{V} = \frac{1000}{3600} \text{ kg.s}^{-1}
\]

Figure 5: Schematic representation of the air flow through the nozzle.

3. Heat transfer in the stream of molten zinc and droplets

First we estimate whether any appreciable cooling of the zinc takes place whilst it
is in a coherent stream in which 1000 kg of molten zinc (at temperature 600°C and 1%
Pb) with density 6430 kg.m\(^{-3}\) flows per hour through a hole of diameter 3 mm. Let the
mean speed be \(\bar{V}\). Mass considerations show that
or $\bar{V} = 6.1 \text{ m.s}^{-1}$. The stream of zinc is observed to break up some 10-15 mm downstream from the hole, so the residence time is about $2.0 \times 10^{-3} \text{ sec}$.

Now Rohsenow et al. (1985, eq. 45, p. 4-37) show that for heat transfer into a thin cylinder of radius $\delta$ in which heat flux through the boundary (and not conduction within the cylinder) is the rate-limiting process, the temperature is given by

$$\frac{T-T_0}{T_a-T_0} = 1 - \exp(-2ht/\delta\rho)$$  (6)

in which $T_0$ is the temperature at time $t = 0$, $T_a$ is the ambient temperature, $h$ is the heat transfer coefficient in Newton’s law of cooling, $\rho$ is the density, and $c$ the heat capacity.

Take the values $\rho = 6430 \text{ kg.m}^{-3}$, $c = 480 \text{ Jkg}^{-1}\text{K}^{-1}$, $h = 57 \text{ Wm}^{-2}\text{K}^{-1}$ (Rohsenow et al., 1985, for forced convective cooling), $\delta = 1.5 \text{ mm}$ (the radius of the stream) and $t = 2.0 \times 10^{-3} \text{ s}$ (the residence time). The exponent $-2ht/\delta\rho$ is approximately $-5 \times 10^{-5}$, and hence $T = T_0$ to an excellent approximation. That is, no appreciable cooling takes place in the stream of zinc.

Similar calculations apply for the time taken for a zinc droplet to solidify. Rohsenow’s formula now becomes

$$\frac{T-T_0}{T_a-T_0} = 1 - \exp(-3ht/\delta\rho)$$  (7)

or, after manipulation,

$$t = \frac{\rho c \delta}{3h} \log_e \frac{T_a-T_0}{T_a-T}$$

We now use the value $h = 6 \text{ Wm}^{-2}\text{K}^{-1}$ (Rohsenow et al., 1985, appropriate for droplets in air flow) and the previous values for $\rho$ and $c$. Thus the time for a droplet at temperature $T_0 = 600^{\circ}\text{C}$ to cool to solidification temperature $T = 420^{\circ}\text{C}$ given an ambient temperature of $T_a = 0^{\circ}\text{C}$ is

$$t = 6.1 \times 10^4 \delta$$  (8)

Therefore a 100 $\mu$ drop cools to solidification temperature in 6.1 seconds whilst a 10 $\mu$ drop takes 0.61 sec. [These time estimates are rather longer than plant experience suggests.] A similar amount of time would be required for surface heat transfer to remove the latent heat of solidification.

4. Droplet stability and collisions

The zone beneath the nozzle is characterised by air motions which are turbulent, with velocity fluctuations from rest to the speed of sound, and with pressure fluctuations of the order of one atmosphere or more. The number of zinc particles produced per second is very large indeed. A spherical particle of diameter $d$ and density $\rho$ has mass $m = \pi d^3/6$. If $10^3 \text{ kg}$ of particles are produced per hour and if we take the values
Atomisation of molten zinc

\[ d = 28 \times 10^{-6} \text{ m and } \rho = 7100 \text{ kg.m}^{-3} \text{ (at 40°C when the zinc is sized)}, \] then the number of particles produced per second is about \( 3 \times 10^9 \). The sheer magnitude of particles which are produced suggests that particle-particle interactions, leading to satellite particles, are likely to be a significant if not dominant part of the process. These ideas are tested by estimates for the number density of droplets, their residence time, and their mean free path before collision with other droplets.

Approximately 1000 m\(^3\) of air (at room temperature and pressure) passes through the atomiser per hour. Equivalently the volume flux per second is 0.28 m\(^3\)s\(^{-1}\). The chamber below the nozzle has a volume of about 0.5 m\(^3\), and thus the residence time of air (including zinc dust and droplets) is 1.8 sec. If \( 3 \times 10^9 \) particles of typical diameter 28 \( \mu \) are produced per second, the average number density \( n \) of particles is

\[ n = \frac{3 \times 10^9}{0.28} = 11 \times 10^9 \text{ particles/m}^3 \quad (9) \]

We apply concepts from ideal gas theory in which molecules of gas move in straight lines until they hit another molecule. The mean free path \( \ell \) of the molecules can be estimated to be (see e.g. Reif, 1965, eq. 12.2.13)

\[ \ell = \frac{1}{\sqrt{2}n\Sigma} \quad (10) \]

where \( \Sigma \) is the cross-sectional area of individual particles. If the cloud of zinc droplets is treated like an ideal gas, \( \Sigma = \pi(d/2)^2 = \pi(14 \times 10^{-6})^2 = 6.2 \times 10^{-10} \text{ m}^2 \). Hence \( \ell \) is approximately

\[ \ell = \frac{1}{\sqrt{2} \times 11 \times 10^9 \times 6.2 \times 10^{-10}} = 0.1 \text{ m} \quad (11) \]

Now individual droplets, by whatever process they may be formed, can be expected to have relative speeds of perhaps 100 m\(^s\(^{-1}\). At these speeds, the time taken to traverse the mean free path is about \( 10^{-3} \) seconds, which is orders of magnitude less than the estimated solidification time for droplets (0.61 seconds for a 10\( \mu \) droplet) and the residence time in the principal collection cavity (1.8 seconds). Multiple collisions between droplets appear to be inevitable!

Brazier-Smith et al. (1973) have studied collisions between droplets and conclude that the combined droplet formed by a collision will break up if the rotational kinetic energy of the coalesced liquid is greater than the surface energy required to re-form the original two droplets. In general, however, our context of collisions between droplets is too complicated to describe mathematically.

Two other simple estimates can be made here. Firstly, for a spherical droplet of radius \( r \) to exist in a stable state, the pressure difference between inside and out is \( \Delta p = 2\sigma/r \) (see e.g. Batchelor, 1967, 1.9.2) where \( \sigma \) is the surface tension. We expect
pressure fluctuations of order 1 atmosphere = $10^5$ Pa and if we take $\sigma$ to be $570 \times 10^{-3}$ Nm$^{-1}$, then the size of the droplets is

$$r = \frac{2 \times 570 \times 10^{-3}}{10^5} = 1.1 \times 10^{-5} \text{m}$$

(12)

This estimate is remarkably similar to the mean droplet size which is observed.

Secondly, Pruppacher & Klett (1978) showed that raindrops of diameter $d$ falling at (subsonic) speed $V$ in air will be dynamically unstable and break up under the action of normal stresses if $d$ is greater than

$$d_{\text{crit}} = \frac{10\sigma}{\rho_{\text{air}}V^2}$$

If we now use the values $\sigma = 570 \times 10^{-3}$ Nm$^{-1}$, $\rho_{\text{air}} = 1.2$ kg.m$^{-3}$ and $d_{\text{crit}} = 28 \times 10^{-6}$ m, the observed mean drop size, we find $V = 410$ m.s$^{-1}$. This is similar to the estimate for the swirling speed $w_2$ found in Section 2, and it indicates that normal stresses play a role in droplet formation.

5. Conclusions

The estimates and considerations presented above lead to the following picture of droplet formation. The axial speed of the air flow through the nozzle is likely to be subsonic at the narrowest point of the nozzle, although supersonic downstream of the nozzle. The swirl velocity is very large at the narrowest point, possibly supersonic. Turbulent boundary layers form along the edges of the nozzle and separate at sharp corners. Prandtl-Meyer expansion fans are expected at these sharp corners. Immediately behind the flat surface of the metal nozzle through which the molten zinc flows, there is a quiescent zone bounded by an inverted cone of rapidly swirling, turbulent air. Somewhere near the apex of this cone, there is a zone (like a hydraulic jump) where the air speed changes from supersonic to subsonic. We call this the transition zone.

As the molten zinc hits the transition zone, it is exposed to violent velocity and pressure fluctuations. The air in the transition zone smashes the molten zinc into droplets. Subsequently, collisions between droplets of molten zinc are likely, thereby producing satellite droplets. In addition, dynamic instabilities due to normal stresses will also break large droplets into smaller droplets. Eventually, the droplets move to a part of the flow where the pressure and velocity fluctuations are smaller, and further breakage no longer takes place. Presumably the droplets solidify in the time during which they are resident in the principal collection chamber, although this is somewhat at variance with estimate (8).
Some general discussion also took place at the Study Group on the likely effect of different nozzle geometries. The discussion was too inconclusive to summarise in this report.

Finally, we mention that further mathematical work with this problem should not be undertaken lightly. The flow takes place in a complex three-dimensional geometry; it is supersonic and swirling, there are undoubtedly turbulent boundary layers that separate from sharp corners, and there is a mixing interface to quiescent air. Further, there is the complication of solidification, collision and dynamic instabilities of droplets. This formidable litany of complications means that detailed mathematical or computational work would be very time consuming. Such work would need to be closely guided by a detailed experimental program incorporating flow visualisation.

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