Chapter 4

Designing Incentive-Alignment Contracts in a Principal-Agent Setting in the Presence of Real Options

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Abstract

We develop a model of incentive compensation for optimal upgrades supplied by an outsourced Information Technology department. We first consider the problem when the rate of technological development is certain and there are no information asymmetries between the parties. We extend this to allow private information between the principal and an agent acting as an external supplier of information technology upgrades. Based on the model in these simple circumstances, we then model uncertain technological improvement, where improvements evolve as geometric Brownian motion, and there is benefit to flexibility in the timing of the upgrade. We are aware of contracts, known as “evergreen upgrades” where a principal pays for upgrades at specified intervals. We find little support for such a contract in our model, and the loss of flexibility in the timing of upgrades is puzzling. The Stern-Stewart problem encourages us to consider just such instances, where contracts limit flexibility that it may be in the interest of both parties to retain. We conclude with a consideration of the wide range of future work needed in this area.

4.1 Introduction

The Stern-Stewart problem description provides a rich context for the development of incentive contracts in the presence of asymmetric information and the need for flexibility. The problem statement allows a broad range of problem definition and solution, and there is substantial scope for further work. For the purposes of this discussion, we will pursue the two general approaches to the problems suggested. These are:

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CHAPTER 4. DESIGNING INCENTIVE-ALIGNMENT CONTRACTS

- Computer upgrade contracts with outsourced IT

- Incentive contracts where there is need for flexibility

These problems share the difficulty that the principal requires private information held by the agent in order to make the appropriate trade-offs for an optimal decision. It is possible that the agent will be unable or unwilling to reveal this information. First, simply, it may be expensive for the agent to (credibly) relay the information to the principal. This would especially be the case for the first problem, where a firm that has decided to outsource information technology. At the time of the outsourcing decision, the principal may be well informed about the technology and its implications for application within the firm. Later, however, as the firm no longer retains the in-house expertise to evaluate technology, the firm may lack the ability to map emerging technological advancements to business objectives.

Second, it may not be in the agent’s interest to reveal information in the most efficient way. Depending on the nature of the compensation agreement, the agent may have an incentive to delay or manipulate the way that verifiable information arrives to the client. The implications of this are described and developed below.

4.2 A Comment on Related Literature in Finance and Economics

This section is intended to provide a brief introduction to the kinds of issues raised in the Stern-Stewart problem, and to provide some reference to the academic work germane to these issues. While option pricing models as applied to real assets has received attention in theoretical work ([5], [18], [19], [9]) there remain persistent concerns that pragmatic difficulties with incentives and compensation have resulted in non-implementation of these models in corporate settings. There is research on the problem of agency generally, and particularly delegation from the headquarters out to divisions, in the development of capital budgeting rules (see [13], [1], [6], [14], [10], [11], [3], [12] and [17], where the latter two consider the effect of influence costs in capital budgeting). There is an extensive literature on the nature of incentive compensation, particularly for CEO’s, and its effect on capital budgeting ([16], [7], [8]), and both theoretical and empirical work has expanded, particularly over controversial bonus and stock-option payouts to CEOs. In this literature, Stern-Stewart’s EVA model (e.g. [15]) plays an important role. Clearly, the Stern-Stewart problem requires significant background to understand the development of related models in finance and economics, and careful application of the results of these models to an understanding of the problem of option value in principal agent problems. In this short report, we can only begin to make progress by narrowing our scope. While a more thorough treatment is certainly warranted, we will have to settle in the time and space allotted for merely beginning the exploration.
4.3 Outsourced Upgrades

Expert advice on upgrade administration is an example of the more general problem of incentive contracts with asymmetric information. At issue is the requirement that for contracts to be enforceable, they must also be verifiable. If there is no need for contract enforcement, then the contract can be written broadly to allow the agent flexibility in the effort levels and timing of the implementation of the contract. If, however, it is in the interest of the agent to choose policies that are, at least potentially, at odds with the interests of the principal, then verifiability is essential for contract enforcement.

We first illustrate the difficulties with flexible contracts in the upgrade problem, and then model the considerations relevant to the optimal solution to the problem.

4.3.1 Upgrades under certainty with symmetric information

We employ a partial equilibrium model for optimal timing of upgrades. The model assumes that exogenous technological improvements lead to greater and greater computing power at constant prices. The firm’s choice is to determine when to upgrade given this exogenous rate of improvement $\alpha$, the firm’s private adoption cost $c$, the contract price $k$ and the firm’s opportunity cost of investment $\delta$. Upgrades occur at discrete times $t_i$, and increase the computing power of the firm by $\alpha (t_i - t_{i-1})$. On any interval where there is no upgrade $[t_i, t_{i+1})$, the benefit is:

$$\int_{t_i}^{t_{i+1}} e^{-\delta t} \alpha (t_i - t_{i-1}) dt$$

So that, if there is no limit to the number of upgrades and the benefit, we have the following:

$$V_0 = \int_0^\infty \int_{t_i}^{t_{i+1}} e^{-\delta t} \alpha (t_i - t_{i-1}) dt$$

Since the benefits of upgrading are exponential and the cost is linear, the firm will instantaneously upgrade for a reasonable set of parameters. For this discussion, however, we limit the upgrade to a discrete choice of upgrade times. We thus seek the optimal conditions for the first upgrade. For that first time interval, the value of the upgrade is measured by the improvement since time $t_0 = 0$, where we normalize the initial computing power to 0. If the cost of the upgrade is $k + c$, and there is value to delaying this outlay, the following equation represents the initial decision:

$$\max_t V_0 = e^{-\delta t} \left( \frac{\alpha t}{\delta} - (k + c) \right)$$

The principal maximizes the value of the firm by choosing the appropriate set of upgrade times $t$. The first order condition provides the optimal for positive $t$ given the non-negativity constraints on the remaining parameters.

$$t^* = \frac{\alpha + \delta^2 (k + c)}{\alpha \delta}$$

Figure 4.1 illustrates the optimal initial upgrade times for discount rates of 15%, 20% and 30% as the exogenous rate of technological improvement increases, other parameters fixed. Higher
discount rates lead to faster adoption rates. This is in part due to the parameterization where technological improvements overcome the benefits from delaying the risk of incurring the cost.

Figure 4.2 plots the optimal adoption times for a range of rates of exogenous technological improvement \( \left( \frac{1}{2}, 1, 2 \right) \) against the discount rate. Clearly, the optimal adoption is more sensitive to the discount rate than to the (linear) rate of technological improvement.

### 4.3.2 Outsourcing with full information

When both parties know the parameters and the nature of the pay-offs, it is straightforward to write an upgrade contract with an outsourced IT department. The optimal delay between upgrades, and the value of the technology at that time, is easily observed. Multiple upgrades in this framework would occur at multiples of \( t^* \), so that \( t_i = it^* \). This result is trivial in the full information case. With information symmetry, we expect there would be little value added by the outsourced IT department, and of course this is where the problem lies.

### 4.3.3 Agency with asymmetric information on upgrades

In the outsourcing decision, the IT supplier becomes an agent of the firm, with a separation of information on model parameters. Once a firm has decided to outsource information technology functions, it is no longer in a position to map the value of the technology improvements to its business needs, so that it can no longer know \( \alpha \). The agent will not know the principal’s private adoption cost \( c \) or its discount rate \( \delta \). The timing of the upgrade will depend upon the incentives in the outsourcing contract.

The principal offers the agent a menu of contracts in order to induce the agent to identify the right time for the upgrade. If this is a one-shot game, then the optimal contract is simply
4.3. OUTSOURCED UPGRADES

![Graph of τ versus discount rate δ](image)

Figure 4.2: τ versus discount rate δ

A fraction of the benefit that the principal receives for the upgrade \( k = \lambda V \), a result that is standard in agency theory. The principal and agent maximize, respectively:

\[
\max_t V_p = e^{-\delta t} (1 - \lambda) \left( \frac{\alpha t}{\delta} - c \right), \quad \max_t V_a = e^{-\delta t} \lambda \left( \frac{\alpha t}{\delta} - c \right)
\]

and the optimal upgrade time is the same for each:

\[
t^* = \frac{c \delta^2 + \alpha}{\delta \alpha}
\]

The payment made is

\[
k = \lambda \frac{\alpha}{\delta^2}
\]

the value of which, at the time of the initial contract, is a benefit to the agent of:

\[
k = e^{(1 - \frac{\alpha^2}{\alpha})} \lambda \frac{\alpha}{\delta^2} > \bar{U}
\]

There are certain constraints on the contract price \( k \). First, \( \lambda \) cannot be too ‘small’. That is, it must compensate the agent for whatever time, effort and materials are required in the execution of the contract (> \( \bar{U} \)). Second, this assumes that principal and agent have the same discount rates. If this is not true, the agent’s optimal exercise will differ from the principal, as illustrated in figure 4.3.

If the contract price is a percentage of the benefit, an agent with a discount rate higher than the principal will prefer to upgrade sooner, and agents with a lower discount rate will upgrade later. This introduces a constraint into the optimal contract. One way to compensate for this is to allow the principal to stipulate a certain level of quality in the contract. For this to be credible,
we require that the principal observe the quality level at time $t$. This is problematic since, if the principal could observe quality, there would be no information asymmetry. Alternatively, we may assume that the principal does observe the value $\alpha t$ with some lag $\Delta t$. In this case, the contract price $k^*_p$, defined as the value of the contract at the principal’s optimal exercise, can be set aside in escrow (perhaps pending suitable verification by an external party) and $k^*_p e^{(\delta_a \Delta t)}$ conveyed after $\Delta t$.

More realistically, we seek an incentive compatible contract that satisfies the constraint for optimal upgrade. We define the relationship between the discount rates as $\delta_a = \delta_p + \epsilon$. Since the agent is likely to be more risk averse than the diversified principal, $\epsilon$ is assumed to be some positive constant. When this is the case, the agent must be paid a bonus for waiting. In this case, the optimal exercise for the agent is $t^*_a = \frac{\delta_a \delta_p + \alpha}{\delta_a}$ and the time that the agent must be induced to wait is $t^*_p - t^*_a = \frac{1}{\delta_p} - \frac{1}{\delta_p + \epsilon}$. Thus the benefit to the agent at time 0 is

$$k^*_a = e^{\left(\frac{\delta_a}{\delta_p} \left(1 - \frac{\epsilon \delta_p^2}{\alpha}\right)\right)} \lambda \frac{\alpha}{\delta_p \delta_a} > \bar{U}$$

which represents a premium over the case where discount rates are symmetric. If we allow $\epsilon < 0$, the principal must then pay a bonus to the agent for early exercise. Figure 4.4 illustrates the amount of the incremental bonus (paid at the time of the upgrade) compared to the ratio of discount rates. The bonus is zero when the discount rates are equal. When the agent’s discount rate is lower than the principal, the agent must be paid a bonus to exercise early. When the agent’s discount rate is higher than the principal’s, a bonus must be paid to delay exercise. This amounts to an ‘information rent’ for the agent.
4.3. OUTSOURCED UPGRADES

Figure 4.4: Agent’s information rent for $\delta_a$ holding $\delta_p$ fixed

4.3.4 Multiple quality dimensions

The Stern-Stewart problem description considers the determination of an optimal policy where there are multiple quality dimensions. For this simple game under certainty, this introduces two difficulties. First, in order to be verifiable, the contract must stipulate how the agent should make trade-offs in the various technologies. For example, suppose that there is a dramatic, unexpected improvement in display technology, and the principal values this highly. How should the agent structure the upgrade contract to offset a slower than anticipated improvement in CPU performance? This will be accomplished much more easily in the case of a linear than a non-linear relationship. More importantly, to the extent that the optimal upgrade time is incentive compatible, the complexities of stipulating precise relationships in the contract can be avoided.

Second, the more detail that is specified, the easier it is to verify compliance with the contract. However, the transaction cost of writing such a contract is likely to require significant time and expense, and this may exceed the value of avoided enforcement. Increased verifiability of contract breach increases the cost of writing the contract while presumably decreasing the cost of contract enforcement.

Multiple dimensions in measuring the quality of the upgrade make the contract more complex in the case where value to the firm cannot be modeled parametrically. If value can be measured, then a linear contract as introduced above will give the right incentives. Nevertheless, the model constraints encountered earlier resurface here. The fact that quality increases along several dimensions does not, by itself, change the incentive compatibility requirements, although it makes writing the contract more complex.

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2This would become considerably more complex when we allow technological improvements to evolve stochastically, in which case implementation would involve multivariate stochastic processes.
4.3.5 Multiple upgrades

Placing the upgrades in a repeated game framework, instead of the one-shot game presented above, also will not solve the fundamental problem. In a repeated game framework, verifiability of the contract with delay $\Delta_t$ will lead to the same kinds of outcomes as the one-shot game. Writing a single contract to cover several upgrades will have the same characteristics, solution and inefficiencies as the one-shot game.

Writing the contracts in sequence introduces new problems. In the one-shot game, the agent’s revelation of the optimal upgrade time in the first period identifies the value of $\alpha$ to the principal. However, knowing that the principal will infer $\alpha$, the agent has an incentive to modify upgrade times as long as it is privately beneficial. If there are other aspects to the outsourcing agreement, however, the agent may provide the principal with optimal times in order to retain future business prospects. To the extent that the agent’s ‘reputation’ for truthful upgrades affects these business prospects, it may be in the agent’s interest to reveal $\alpha$ early on.

This latter point relates to the Stern-Stewart problem more generally, in that much of the concern over the agent’s private incentives are less important in the context of an on-going relationship with the principal. So, for example, if the agent has a number of contracts with the principal, not just over upgrades, there are incentives for truthful revelation if the principal will eventually observe product quality. The greater the lag $\Delta_t$ the less important is this incentive, but it remains a potential rationale for outsourcing a number of activities to the agent.

4.3.6 Summary points on upgrades under certainty

This section has presented a simple upgrade problem in the context of constant, linear exogenous quality improvements. The optimality conditions in this setting are straight-forward, and allow a simple contract to cover the transaction. In contrast, asymmetric information significantly complicates the model. Differences in risk preferences between the risk-averse agent and the risk-neutral principal impose constraints on the model. While we have model-led a constant $\lambda$ and adjusted the value of payment, it would be helpful to consider optimal values of $\lambda$ for a range of discount relationships $\frac{\Delta}{\delta}$. While more work on the simple model could certainly extend it, we will devote our efforts instead to the development of a similar problem where the upgrade occurs under uncertainty.

4.4 Upgrades Under Uncertainty

The simple model of linear technological innovation in the prior section allows for straight-forward solution of the problems in information asymmetry. In this section, we turn to the more complex, and more to the point, more in line with the Stern-Stewart problem, model of technological evolution in computing as a diffusion\(^3\) process. Assume that the value of the

\(^3\)A more sophisticated model may be developed that includes jump processes for technological discontinuities and dramatic changes in the various components of a computer system. These jumps may be either positive or negative shocks. An example of a positive shock is the development of new algorithms for data compression that improves the quality of the transfer. A negative shock would be an announcement that the developer of a certain computer system would no longer produce or maintain hardware.
underlying technology evolves according to the following equation of motion:

\[ dP = \alpha P + \sigma P dW \]

Where \( P \) is the value, \( \alpha \) the drift and \( \sigma \) the volatility of the technology process.

If there is a perfectly competitive market to supply the technology, then the value at time \( t \), denoted \( P_t \), and the cost of acquiring the technology at that time, denoted \( K_t \), will be equal. As long as this is the case, the principal can upgrade according to whatever schedule it prefers, exchanging the cost of adoption for the value of the technology. In perfectly competitive markets, a myopic upgrade policy will do.

A perfectly competitive market structure appears to trivialize the problem, but this result assumes that the effects of information asymmetry are competed away in a market with free entry and exit. The Stern-Stewart problem description identifies concern over the loss of the option to delay. In order for there to be value to delay, it must be that the current technology is not appropriately priced to the firm. For example, the assumption that adoption at a fixed date is suboptimal implies that the asset is overpriced in comparison to the firm’s own valuation of expected availability of future quality. If this valuation were the same for all adopters, the technology would not trade until it had declined to this lower price.

If trade is based on the firm’s private adoption cost, then a range of prices may exist in the market. We may model the strike price as the contract price plus the firm’s private adoption cost: \( K_t = k_t + c_t \), where \( k \) is the price on delivery of the contract and \( c \) is the adoption cost. The firm’s objective is to gain maximum value from the contract:

\[ \max V_t = P_t - K_t \]

### 4.4.1 Information and transaction costs

The Stern-Stewart description of an evergreen upgrade, however, suggests a deeper problem, subject to transaction uncertainty. The principal pays a sum at time \( t_0 \) for the upgrade of computer systems at a future date, denoted \( T \). At \( T \), the agent receives \( k_T \) to provide a computer that is “upgraded to the latest standards” for the principal. In terms of financial contract, this is similar to a forward contract, usually written for standardized commodities. In contrast to standard forward contracts, however, the evergreen contract does not specify the quality of the product at the date of delivery, and so is particularly subject to ex post opportunism. For example, establishing “latest standards” in computing is problematic, and is especially of concern to the principal if the agent is facing liquidity constraints as the upgrade date approaches.  

\footnote{Why anyone would enter such a contract is puzzling to us, yet this is precisely the kind of agreement made at some university computing centers. We have heard it argued that this contract allows the upgrade provider the ability to secure the necessary resources at the time of the upgrade. The implication is that giving the agent commitments to timing allows coordination benefits that compensate for the lack of flexibility in timing. At issue is an estimate of the principal’s loss due to the agent’s inability (due to lack of resources) to upgrade versus the loss of option value due to the contract commitment. Without some estimate of the probability and magnitude of these losses, we are not in a position to comment on the merits of the “evergreen” contract. However, suitable extensions to the model should provide some reasonable estimate of the trade-offs.}
If the model were played as a one-shot game, the principal would risk contracting with an unscrupulous agent where moral hazard would result in the agent purchasing the minimum acceptable “latest standard” computer, and pocketing the difference. In a repeated game framework, however, the agent may supply the requisite quality if future profits depend upon contract compliance. These results are standard in game theoretic models of repeated interaction, such as the cooperation observed in repeated “prisoner’s dilemma” games (see [2]).

In this framework, however, there is concern over the loss of flexibility in the timing of the upgrade decision. There are two potential concerns. One is that the evergreen program stipulates the timing of the upgrade well in advance of realizing the value of it. Clearly, flexibility in the timing of the upgrade is important if technological evolution is uncertain. In the case of upgrades under certainty, we saw that the contract could be written for any number of upgrades with a specified lag between upgrades. When there is uncertainty because technological innovation is a diffusion process, the principal may prefer to delay an upgrade when the contracted time arrives. That is, the European option would add value to a pre-specified contract time. But second, an American option would add even greater value than the European option, since it provides greater flexibility about the timing of exercise. It is a standard result in option pricing that the greater exercise flexibility of the American option implies that the European option is a lower bound for its value. Adding these flexibilities to the contract could only increase the value of the upgrade decision.

4.4.2 Asymmetric information and value uncertainty

We next present a model of the delegation of the upgrade decision under asymmetric information. The agency concerns under value uncertainty in an outsourced model are not fundamentally different from the decision presented above. Exercise at the optimal will have similar value and verifiability constraints, and the fact that $P$ evolves stochastically, while complicating the picture, does not appear to fundamentally alter the nature of the contract. That is, if we allow the agent to share in the profitability of the contract, the agent will have incentives to execute the upgrade contract at the time that is optimal to the principal. For this reason, we next consider how the upgrade must be structured with an in-house IT department. This analysis will provide at least some insight into what it may make sense to outsource the upgrade decision, since, as we shall see, there are important difficulties in making the upgrade efficiently. In this context, then, principal refers to the headquarters of the organization, and agent refers to the IT department.

We make the following assumptions about information held by the principal and the agent. 1) At the time that an opportunity becomes available, both the principal and the agent are aware of the initial value of the upgrade, and the drift and volatility. These are denoted $P_0$, $\mu$ and $\sigma$. 2) After hiring the agent, it is assumed to be too expensive for the principal to continue monitoring the value of the upgrade; $P_t$ is in only the agent’s information set. The upgrade is undertaken when the agent recommends, and this is denoted time $T$. The value of the upgrade at exercise, $P_T$ is known by both the principal and agent. The model assumes that both the principal and the agent are aware of the optimal conditions to exercise the option, but only the agent has private information on whether those conditions are satisfied. The agent’s task is to monitor the value process, and announce to the principal when to upgrade.
4.4. UPDATES UNDER UNCERTAINTY

4.4.3 Payoffs

The upgrade date, denoted $T$, is not known at the start of the contract. As before, subscripts $a$ refer to the agent, $p$ to the principal. The model assumes that the principal discounts at the risk-free discount rate during the wait-time, and the agent’s discount rate is greater than that: $\delta_a \gg \delta_p$.

Prior to exercise, the value of the upgrade $P_t$ evolves stochastically according to geometric Brownian motion. The principal’s cost at the time of the upgrade is $K = k_t + c_t + b$. The terms are as before, though we have added a bonus $b$, and we assume for simplicity that $k, c$ are fixed over the relevant time horizon. The value of the option to develop the opportunity is denoted $V = \max(P - K, 0)$. There is an additional constraint to pay the IT employee a salary $s$. Under these conditions, the expected value of the upgrade at the start of the contract is defined as:

$$\pi_p = e^{(-\delta_p T)} (V - K) - s \frac{1 - e^{(-\delta_p T)}}{\delta_p}$$

4.4.4 Simple model of agent’s pay-offs

It is useful to understand the timing decision if it is made inside the firm. Constraints on compensation require that the agent be paid a salary $s$ plus a bonus $b$ at exercise. We define the agent’s objective function as the utility from the compensation package. The agent’s risk-aversion is characterized by a higher discount rate over the project development horizon. Under these conditions, the risk-aversion is embedded in the agent’s demand for a higher premium on future cash flows, making the agent more impatient than the risk-neutral principal. We seek the right compensation package to offer the agent, since the principal seeks someone who will make the exercise decision consistent with the principal’s preferences.

$$\pi_a = e^{(-\delta_a T)} b + s \frac{1 - e^{(-\delta_a T)}}{\delta_a}$$

At this point, the compensation does not assure that the agent will act in the principal’s interest, or even that the agent will want to enter into the contract. If the principal offers a choice between a fixed bonus or a continuous salary, the agent will choose either to exercise immediately and get the bonus ($T = 0$), or never to exercise and collect the salary ($T = \infty$). This will be a function of the agent’s discount rate $\delta_a$, the bonus $b$ and the salary $s$. The agent will take the salary in perpetuity unless $\frac{s}{\delta_a} < b$, in which case, the agent will take the bonus immediately.

In either case, the return to the principal will not depend on optimal exercise, and thus the agent offers no value. The principal has several options. First, she may offer a bonus that is a linear function of her optimal, denoted $V^*_p$. If salary is set $s = 0$, the agent will have an incentive to maximize the value of the option. The optimal exercise for the agent is denoted $V^*_a$. Under these constraints, the model assumes certain regularity conditions that yield the following results. First, the optimal value of the option is a function of the volatility of the process ($\sigma$), the drift in the process ($\mu$), the discount rate ($\rho$), and a convenience term that represents the difference between the discount rate and the drift ($\delta$). There are also restrictions
on the parameters for solutions finite in $T$.

\[
dV = \mu V \, dt + \sigma V \, dz
\]

\[
\delta = \rho - \mu
\]

\[
\mu < \rho
\]

The following value-matching and smooth-pasting conditions are optimal:

\[
A(P^*)^\beta = \frac{P^*}{\delta} - K
\]

\[
\beta A(P^*)^{(\beta-1)} = \frac{1}{\delta}
\]

In order to satisfy these, $\beta$ must solve the following quadratic:

\[
\frac{1}{2} \sigma^2 \beta (\beta - 1) + (\rho - \delta) \beta - \rho = 0
\]

and if $\beta > 1$:

\[
\beta = \frac{1}{2} - \frac{\rho - \delta}{\sigma^2} + \sqrt{\left(\frac{\rho - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{\rho}{\sigma^2}}
\]

So, for given $K$, $\rho$, $\alpha$, $\sigma$ we can find an optimal exercise condition.

\[
P^* = \frac{\beta \delta K}{\beta^2 - 1}
\]

Optimal exercise will differ according to risk-preferences, modeled as differences in discount rates $\rho$. Define $\delta$ strictly positive as follows:

\[
\delta_p = \delta_p - \mu - \sigma^2 > \delta_a = \delta_a - \mu > 0
\]

\[
\beta_p = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{\delta_p}{\sigma^2}}
\]

\[
\beta_a = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{\delta_a}{\sigma^2}}
\]

Clearly, $\beta_p < \beta_a$ since $\delta_p < \delta_a$, and, applying this to $V_a$ and $V_p$, the following are the optimal strike prices:

\[
V_a = \frac{\left(1/2 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 1/2\right)^2 + 2 \frac{\delta_p}{\sigma^2}}\right) K}{-1/2 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 1/2\right)^2 + 2 \frac{\delta_p}{\sigma^2}}}
\]

\[
V_p = \frac{\left(1/2 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 1/2\right)^2 + 2 \frac{\delta_p}{\sigma^2}}\right) K}{-1/2 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 1/2\right)^2 + 2 \frac{\delta_p}{\sigma^2}}}
\]

These results demonstrate that $V_p > V_a$ since $\delta_p > \delta_a$, implying that the agent will exercise prematurely in comparison to the principal’s optimal. Figure 4.5 depicts the divergence between the principal and the agent’s optimal exercise as a function of the volatility of the project.
4.4. UPDATES UNDER UNCERTAINTY

![Graph showing the relationship between principal and agent value versus σ.]

Figure 4.5: Principal and Agent value versus σ

4.4.5 Results and implications

If the principal is restricted to the choice of salary plus bonus, the optimal compensation contract must account for both the incentive to reveal information and the reservation wage of the agent. As the model implies, the bonus must be constructed as a function of the difference between the discount rates, the volatility of the underlying price, and the drift of the project. If the bonus is constructed as \( \lambda (V_p - V_o) \) with \( \lambda > 0 \) the agent may exercise at the appropriate time, but a similar constraint as under uncertainty must compensate the agent for 1) the expected wait and 2) the difference in discounting. The solution is not fundamentally different than the bonus under certainty. The incentive must give the agent sufficient reason to wait until the price process reaches the smooth-pasting condition, and for this we require a combined salary and bonus. If \( s \) compensates the agent for precisely the reservation wage of waiting, then the bonus\(^5\) will be adequate to induce the agent to wait.

If the principal can impose a sufficient penalty on the agent for early exercise, and if the value of the project becomes common knowledge at exercise, then the agent can be induced to exercise at the principal’s optimal point. Without the ability to charge a penalty, the bonus must satisfy the constraint that the discounted value of the bonus plus the periodic compensation is sufficient to induce the wait. By relating the bonus to the principal’s optimal strike, we give the agent an incentive to maximize the bonus.

\(^5\)In a linear compensation scheme with only a bonus payment that is a constant proportion of the value at exercise, this would be resolved.
4.4.6 Complications
These results assume that the principal would never choose to abandon the upgrade. While
the march of technological progress has lead to decade after decade of upgrades, this need
not continue to be the case. At some point, there may be no rationale for the principal to
hire the agent to monitor technology. In such a case, the principal would like the agent to
abandon monitoring, but, because the agent is compensated for waiting, the agent may have
no incentive to inform the principal to abandon. Under the kinds of diffusion processes that we
have examined, the principal may form expectations about when to expect an announcement of
optimal upgrade time, however given sufficient volatility in the process, there may be significant
uncertainty and room for inefficient delay. Alternatively, in a jump-diffusion process, sudden
increases or decreases in the value of the upgrade may make the need sudden abandonment hard
to detect.

In such a situation, the principal must pay the agent an abandonment bonus. We leave
the construction of such a bonus for later work, but in recognizing the need for such a bonus,
we determine at least one of the benefits of a linear compensation scheme. If the agent’s
compensation is based on a share of the profits at the time of the upgrade, both parties will
abandon at the optimal time. This leads to inevitable difficulties in incentives if the costs are
shared by the parties, however. In a debt-like arrangement, where the principal pays for project
continuance, the agent has an incentive to avoid abandonment on the chance that things may
turn around. In an equity-like arrangement, where both parties share the costs of continuation,
the agent’s incentives will be aligned with the principals.

4.5 Summary and Conclusions
The results from this preliminary model are suggestive of why firms may outsource the IT
function. The models under certainty demonstrate efficiencies in contracts that allow sharing.
There are a number of practical reasons why these contracts may be hard to write or enforce.
We have emphasized the difficulties that the principal has in monitoring the agent, and only
referred to the problems that the agent may have in observing the principal’s private valuation
of the technology. Verifiability is an important problem for both players in this game.

The Stern-Stewart problem description allows for a much more sophisticated treatment
of the problems in flexibility and contracts. While the treatment of the literature in this report is
brief, there is a great deal more in economic models that allow for renegotiation of contracts.
For example, some models allow the principal to sell the business to the agent (see [4]). This
will clearly make more sense in other contexts than upgrades, but it does suggest an approach to
the development of models where contingent claims allow for flexibility in contract enforcement.
While there is much more to do, this report provides a framework for beginning, and it is hoped
that much of the work referenced here can be applied to extensions of the models.
Bibliography


[9]


